Study of aerodynamical behaviour and serviceability limit state of suspension footbridges under wind action

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ABSTRACT: An aerodynamical analysis of four existing light footbridges of different measures of making span of footbridges stiff enough (i.e. inclined cable system, horizontal truss, horizontal arches) is presented. A model of wind action adopted in agreement with a quasi-steady concept takes into consideration not only unsteady air onflow but also motion of the structure itself. Wind action caused by vortices is neglected. In addition, it is assumed, that considered structures behave in a linear elastic way.

KEYWORDS: Suspension footbridges, wind action, horizontal and torsional stiffness, statical and dynamical response.

1. THEORETICAL APPROACH TO A PROBLEM

An analysis of four existing light footbridges is made at assumptions as follows :

- Wind action on pylons and cable systems of footbridges is of static-type;
- Wind action on spans of footbridges is considered in accordance with a quasi-steady theory [1, 5].
- Linearised small vibrations of footbridges around their mean (static) position are considered.

Quasi-steady theory enables to consider several aerodynamic phenomena, such as galloping, special type of flutter, buffeting, and divergence. Three components of wind action include static and dynamic wind actions on spans of footbridges. These three components of wind action, i.e. drag force, lift force and aerodynamic moment, can be given by :

$$q = \rho(\bar{u} + u'_s)^2 / 2 \tag{1}$$

$$W = (\upsilon'_s + \dot{\xi}\sin\bar{\alpha} - \dot{\eta}\cos\bar{\alpha})/(\bar{u} + u'_s) - [x_G\cos(\bar{\alpha} - \varepsilon) + y_G\sin(\bar{\alpha} - \varepsilon)]/(\bar{u} + u'_s)\dot{\varepsilon} - \varepsilon \tag{2}$$

$$w_x = q\,D\,[C_x + C_{xy}\,W];\ w_y = q\,D\,[C_y + C_{yx}\,W];\ w_m = q\,D^2\,[C_m + C_{mm}\,W] \tag{3}$$

where: ρ - density of air; $\bar{u}$, u', υ', $\bar{\alpha}, \alpha'$, ξ, η, ε – as in Figure 1; D – characteristic dimension of cross-section (height or breadth); x_G and y_G – coordinate of geometric centre of the outline curve of cross-section; C_x , C_{xy} , C_y , C_{yx} , C_m, C_{mm} – aerodynamic coefficients; $\bar{u}, \bar{\alpha}$ – time averaged quantities in time period T=10 min; $\dot{\xi}, \dot{\eta}, \dot{\varepsilon}$-derivatives with respect to time of the respective quantities; u'_s, υ'_s – fluctuations of spatially averaged quantities in a surface domain characteristic for a given structure cross-section (e.g. domain of surface area $S = \kappa \cdot D \cdot 1$ at a distance, at least, D in front of a structure cross-section, where $\kappa \cdot D$ corresponds to the width of the cross-section structure wake).

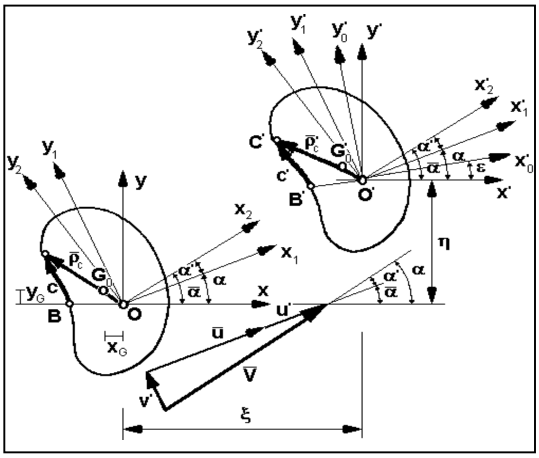

Figure 1. Systems of coordinates and relations between different geometrical quantities in the case of a moving structure and an unsteady air onflow

2. MATHEMATICAL MODEL OF THE THREE DEGREES OF FREEDOM SYSTEM AS A SUBSTITUTIONAL SYSTEM OF EXISTING STRUCTURE

Spans of the considered footbridges should be treated as slender structures, which can be divided into *m* elements. Each of them is represented by the inner point *k* of the cross-section (see Figure 2).

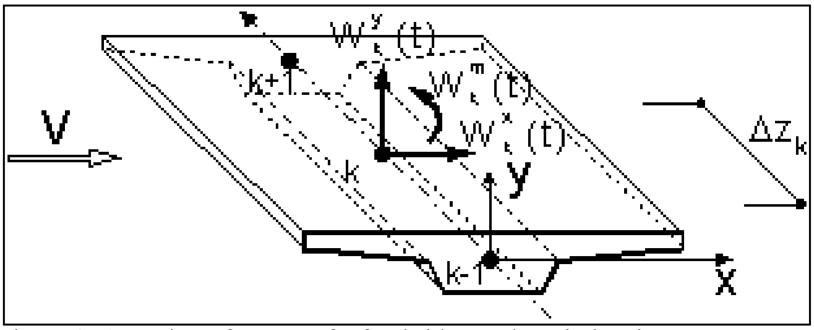

Figure 2. A section of a span of a footbridge under wind action

Formulas (1-3), which are related to all points of a span next are transformed to the global system of coordinates. It is assumed, that the structures behave in a linear elastic way. Their motion can be described by a matrix equation:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\}=\{W\} \tag{4}$$

The Bubnow-Galerkin's method is adopted. As a result, a system of many degrees of freedom is substituted by a representative system of three degrees of freedom. Motion of the substitutional system under wind action described by the formulas (1-4) can be expressed by:

$$M_j \ddot{\Psi}_j(t) + C_j \dot{\Psi}_j(t) + K_j \Psi_j(t) = \sum_{k}^{m} (X_k^j \{A(t)\}_k^{\xi T} + Y_k^j \{A(t)\}_k^{\eta T} + \Phi_k^j \{A(t)\}_k^{\varepsilon T})\{\chi(t)\}_k \tag{5}$$

$$\{A(t)\}_k^{\xi T} = [C_x W_k(t),\ C_{xy} W_k(t) a_k(t),\ C_{xy} W_k(t) b_k(t),\ -C_{xy} W_k(t) c_k(t),\ -C_{xy} W_k(t) d_k(t),\ C_{xy} W_k(t) e_k(t),\ -C_{xy} W_k(t)] \tag{6}$$

$$\{A(t)\}_k^{\eta T} = [C_y W_k(t),\ C_{yx} W_k(t) a_k(t),\ C_{yx} W_k(t) b_k(t),\ -C_{yx} W_k(t) c_k(t),\ -C_{yx} W_k(t) d_k(t),\ C_{yx} W_k(t) e_k(t),\ -C_{yx} W_k(t)] \tag{7}$$

$$\{A(t)\}_k^{\varepsilon T} = [DC_m W_k(t),\ DC_{mm} W_k(t) a_k(t),\ DC_{mm} W_k(t) b_k(t),\ -DC_{mm} W_k(t) c_k(t),\ DC_{mm} W_k(t) d_k(t),\ DC_{mm} W_k(t) e_k(t),\ -DC_{mm} W_k(t)] \tag{8}$$

$$\{\chi(t)\}^T{}_k=[1,\ 1,\ \dot{\xi}_k(t),\dot{\eta}_k(t),\dot{\varepsilon}_k(t),\dot{\varepsilon}_k(t)\varepsilon_k(t),\varepsilon_k(t)] \tag{9}$$

$$W_k(t) = D\rho u_{sk}^2\,\Delta z_k \tag{10}$$

$$u_{sk} = u_s(z_k,t) = \bar{u}(z_k) + u'_s(z_k,t) \tag{11}$$

$$a_k(t) = \upsilon'_s(z_k,t)/u_s(z_k,t) \tag{12}$$

$$b_k(t) = \sin\bar{\alpha}(z_k)/u_s(z_k,t) \tag{13}$$

$$c_k(t) = \cos\bar{\alpha}(z_k)/u_s(z_k,t) \tag{14}$$

$$d_k(t) = [x_G\cos\bar{\alpha}(z_k) + y_G\sin\bar{\alpha}(z_k)]/u_s(z_k,t) \tag{15}$$

$$e_k(t) = [y_G\sin\bar{\alpha}(z_k) - x_G\cos\bar{\alpha}(z_k)]/u_s(z_k,t) \tag{16}$$

where : $\Psi_j(t)$ – j-th generalized coordinate; $X_k^j,\ Y_k^j, \Phi_k^j$ – translation and rotation coordinates of j-th free vibration mode of the point which represents k-th structural element; m- number of structural elements; $M_j,\ C_j\ K_j$ – generalized mass, damping and stiffness, Δz_k – length of k-th structural element.

The complex form of the non-linear set of differential equations is resolved in a numerical way. The Newmark method of integration of the motion equations is adopted. The algorithm of the Newmark method is based on the own computer programme which gives the general displacements ξ, η, ε for each time step in k representative points for particular structural elements.

3. DESCRIPTION OF THE ANALYSED STRUCTURES

Among many means of making spans of light footbridges stiff enough, three different solutions are presented and analysed. First type of footbridge is a suspension footbridge in Piwniczna (see Figure 3), which has vertical truss and two additional horizontal steel arches. Moreover, horizontal stiffness is amplyfied in adition by steel ortothropic plate.

Second type of footbridge are suspension footbridges in Rożnów (see Figure 4) and in Tropie (see Figure 5). In these footbridges transversal and torsional stiffnesses are ensured mainly by inclined cable systems.

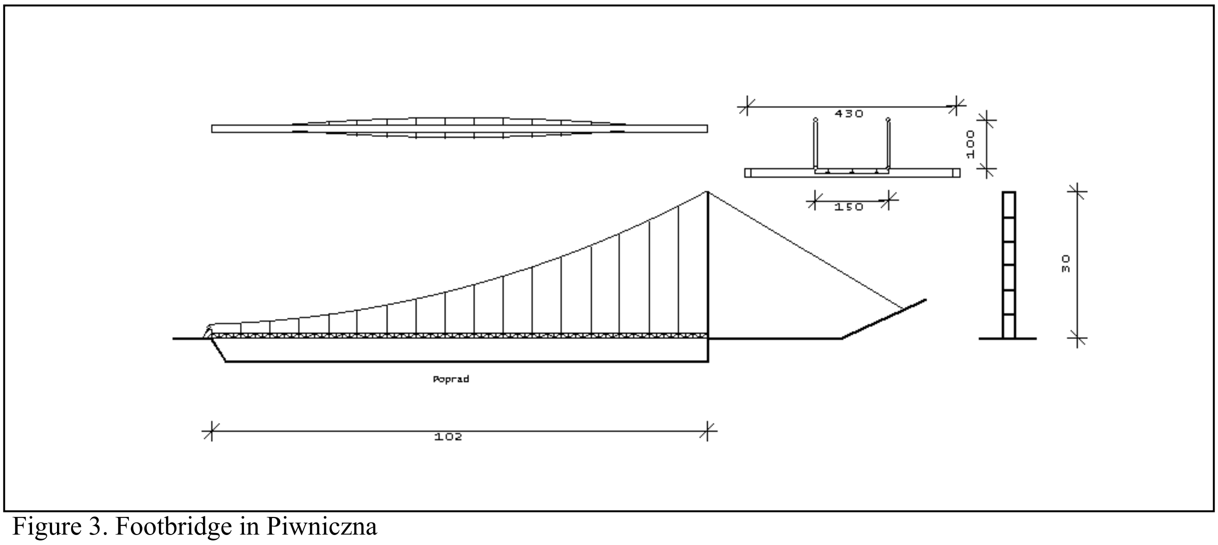

Figure 3. Footbridge in Piwniczna

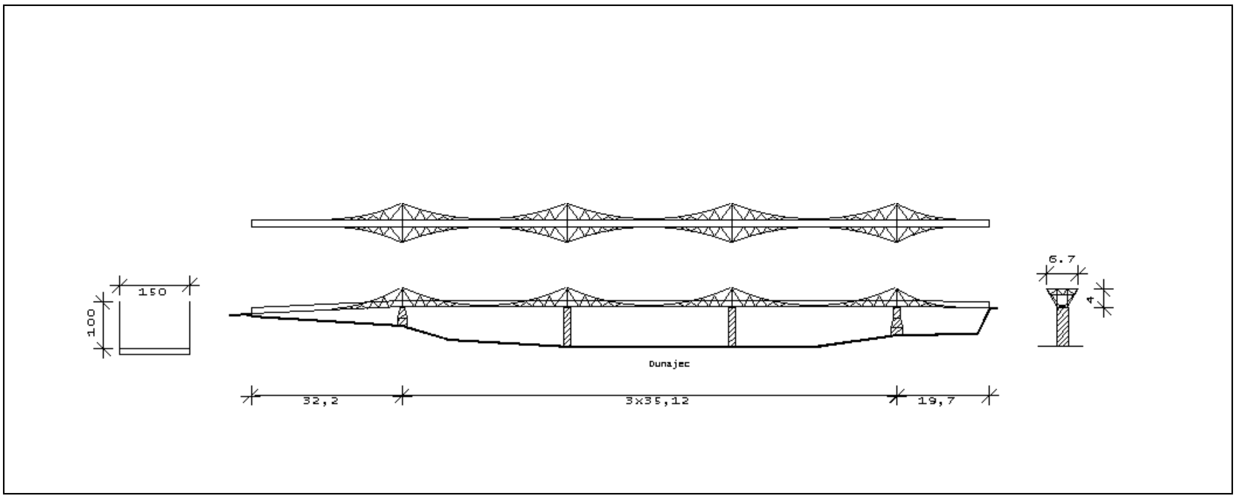

Figure 4. Footbridge in Roznów

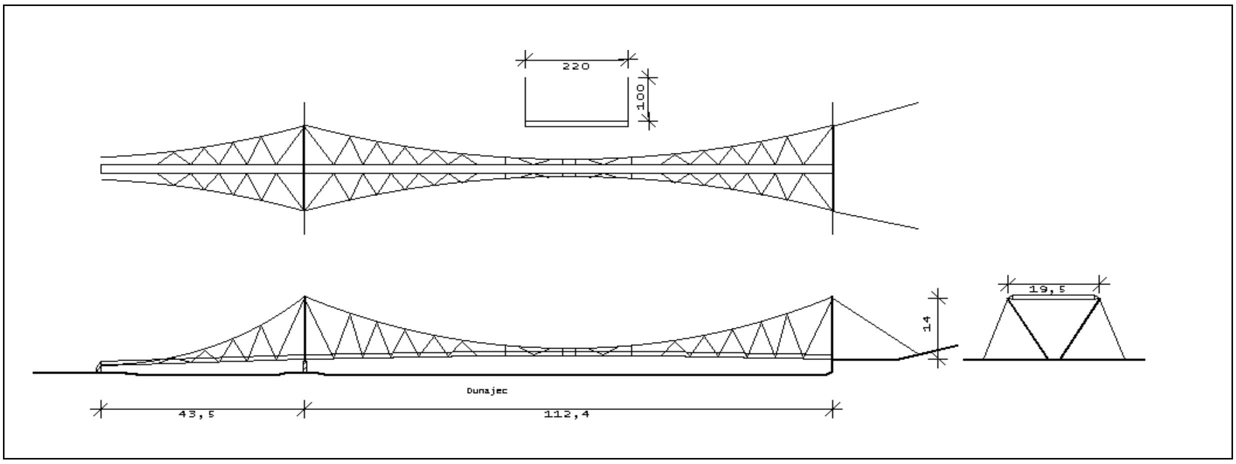

Figure 5. Footbridge in Tropie

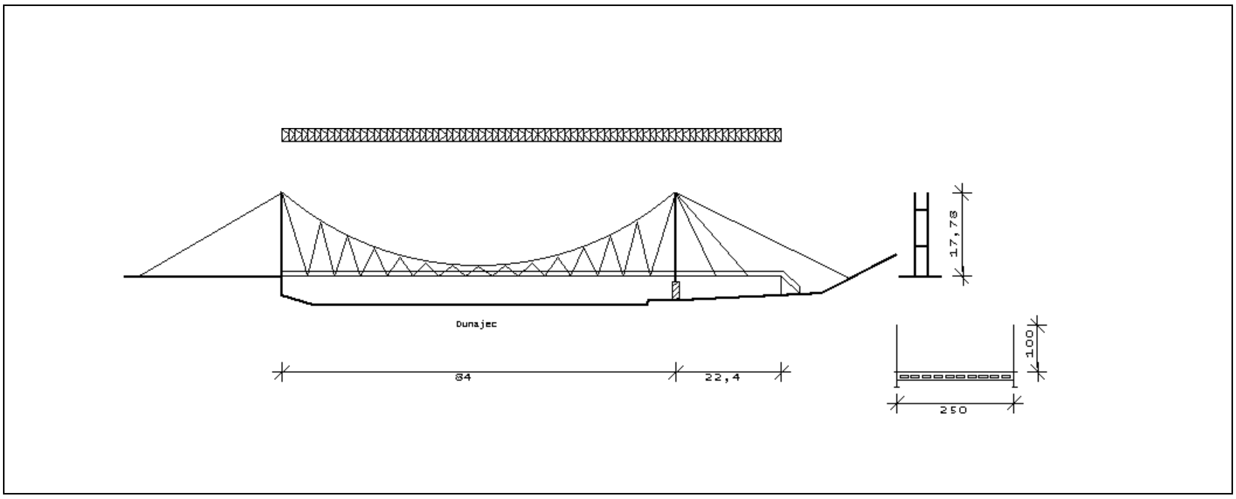

Figure 6. Footbridge in Tylmanowa

Third type of footbridge is a suspension footbridge in Tylmanowa (see Figure 6). Horizontal stiffness of this footbridge is mainly ensured by horizontal truss, what is the most often case encountered in practice. Torsional and vertical stiffnesses come from suspension cables and hangers.

4. EXEMPLARY CALCULATION RESULTS

4.1 *General assumptions*

The following general assumptions have been assumed in numerical calculations:

- Linearised small vibrations of footbridges, around their static position, are considered. This means, that vibrations of footbridges influenced by static component of wind action, come in question. Because of this, a character of vibration modes can be complicated and not so „clear” (vertical, horizontal etc.), but of a spatial character (quasi-vertical, quasi-symmetrical, etc.).
- With respect to horizontal, vertical and torsional directions, the lowest three symmetrical or quasi-symmetrical vibration modes, showed in tab. 1, are considered.

Table 1. Free vibration modes and eigenfrequencies of the footbridges

Footbridge	Frequencies [Hz] and modes					
Piwniczna	H T S	H T S	H T A	V S	H T A	V A
	0.588	0.685	0.936	1.156	1.326	1.404
Roznów	H T A	H T A	H T S	V S		
	0.610	0.638	0.692	1.006		
Tropie	H T S	H T V S	H T A	H T A		
	0.500	0.813	0.871	1.223		
Tylmanowa	H S	1.803	V S	T S		
	1.215	H S	1.862	1.959		

S – symmetrical or quasi-symmetrical mode, A – asymmetrical or quasi-asymmetrical mode, H – horizontal or quasi-horizontal mode, V – vertical or quasi-vertical mode, T – torsional or quasi-torsional mode. Classification of the vibration modes of the footbridges is rather complicated

4.2. *Data assumed in calculations*

- Wind characteristics :
  - wind is simulated by WAWS method;
  - von Kármán spectrums of three components of wind velocity are adopted (k = 0.002, z_o = 10 m);
  - scale of turbulence is equal to 100 m;
  - power wind profile is assumed (α = 0.24);
  - mean wind velocity 10 m above ground $\bar{u}$ = 20 m/s.
- Aerodynamical coefficients for footbridges in Rożnów, Tropie and Tylmanowa are the same, assumed as for a flat plate, so the aerodynamical coefficients are as follows :
  $C_x = 0.281$; $C_{xy} = 0.052$; $C_y = 0.000$; $C_{yx} = 4.561$; $C_m = 0.013$; $C_{mm} = 0.212$.
  Footbridge in Piwniczna has vertical trusses, because of this aerodynamical coefficients are as follows:
  $C_x = 2.000$; $C_{xy} = 0.052$; $C_y = 0.000$; $C_{yx} = 4.561$; $C_m = 0.659$; $C_{mm} = 0.315$.
- Logarythmic decrement of damping $\Delta = 0.040$.

4.3. *Response of the structures*

For the each footbridge mean, maximum and r.m.s. value of horizontal, vertical and torsional displacements have been obtained. In analysis of aerodynamic behaviour of footbridges, several non-dimensional parameters have been used:

- Ratio of the static (average; ξ_S) and amplitude (dynamic; ξ_A) horizontal displacement to the length of the span;
- Ratio of the static (η_S) and amplitude (η_A) vertical displacements to the length of the span;

- Angle of the static, (ε_S) and amplitude (ε_A) torsion of the span (in radians);
- Ratio of the death-weight to the imposed action γ.

Values of these parameters are given in table 2.

Table 2. Non-dimensional parameters

Footbridge	γ	Static displacements			Amplitudes		
		1000 ξ_S	1000 η_S	ε_S [rad]	1000 ξ_A	1000 η_A	ε_A [rad]
Rożnów	0.165	2.414	0.367	0.070	0.861	0.071	0.030
Tropie	0.101	1.010	0.353	0.059	0.063	0.009	0.004
Tylmanowa	0.312	0.047	0.000	0.000	0.018	0.066	0.000
Piwniczna	0.372	1.031	0.010	0.009	0.345	0.142	0.035

These four footbridges can be divided into two groups: with inclined cable system (for example Tropie) and without inclined cable system (for example Piwniczna; the same shape for Tylmanowa).

For footbridges in Piwniczna and in Tylmanowa dependence of the horizontal displacement on the static horizontal aerodynamical force is linear character, for footbridges with inclined cable system is of nonlinear character.

Moreover, character of the movie of middle part of span is different for these two types footbridges (comp. figures 7÷10).

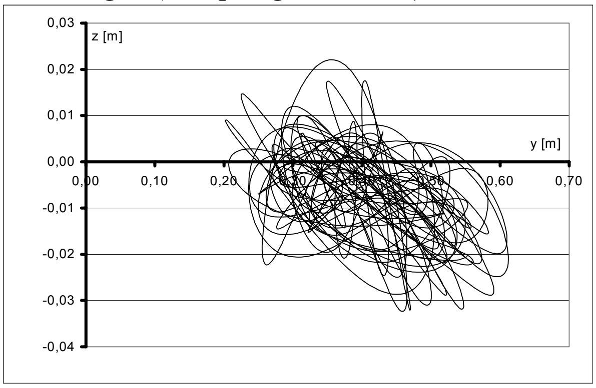

Figure 7. Planar motion of the central point of span of Piwniczna footbridge.

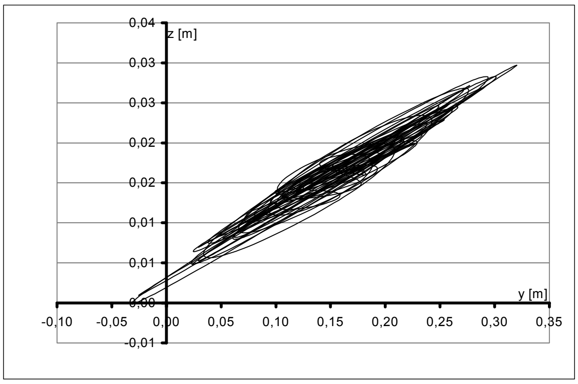

Figure 8. Planar motion of the central point of span of Rożnów footbridge.

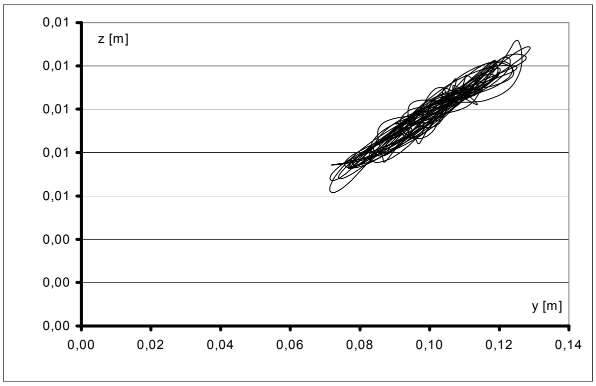

Figure 9. Planar motion of the central point of span of Tropie footbridge.

Figure 10. Planar motion of the central point of span of Tylmanowa footbridge.

4.4. *Serviceability limit state*

To evaluate of the serviceability limit state of the analysed footbridges, following standards and proposal have been taken into account:

- Polish Standard [6]:

According to the Polish Standard values of vertical statical displacement of span and a period of horizontal vibrations should be as follows:

$$\eta_S < 0.0026; \;\; T_H < 0.1L \qquad (17)$$

where L -length of the span [m]; T_H - period of the first natural horizontal vibrations [s].

- Authors's proposal with respect to angle of torsion of span:

In winter months span of the footbridges can be ice-coated. Because of this, angle of the torsion of the span can't be too big; so our proposition is as follow:

$$\varepsilon_S < 3^o \qquad (18)$$

- Flaga's proposals [7] (in which ISO recommendations [8] are included).These proposals in form of dependencies between rms of acceleration of span and frequencies of vibrations are presented on Figures 11 and 12 together with numerical results, obtained for the analysed footbridges (1 - Piwniczna, 2 - Rożnów, 3 - Tropie, 4 - Tylmanowa). Acceptable values are placed below solid lines.

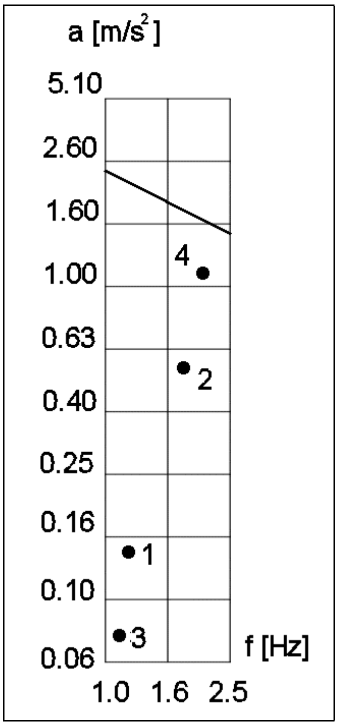

Figure 11. Acceptable curves and numerical results for vertical vibrations

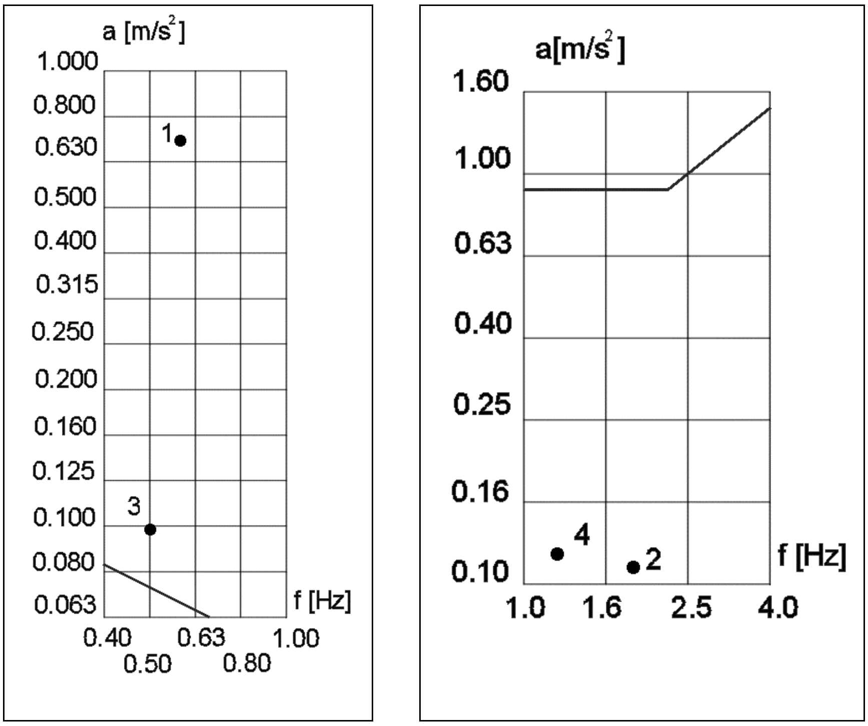

Figure 12. Acceptable curves and numerical results for horizontal vibrations

- Moreover, another requirements with respect to not allowable frequency bands for footbridges are recommended by authors.

For footbridges vibrations caused by pedestrian excitation can be dangerous for the following horizontal, vertical and torsional frequencies bands:

$$f_H \in (1.9 \div 2.1)\ Hz;\ f_V \in (1.65 \div 2.35)\ Hz;\ f_T \in (1.65 \div 2.35)\ Hz \quad (19)$$

Results of analysis of serviceability limit state for the considered footbridges are presented in Table 3.

Table 3. Results of analysis of serviceability limit state for considered footbridges

Footbridge	Stiffness and statical displacement requirements			Dynamic requirements		Excitation by pedestrian			All
	ξ	η	ε	ξ	η	ξ	η	ε	
Piwniczna	Y	Y	Y	N	Y	Y	Y	Y	7/8
Rożnów	Y	Y	N	N	Y	Y	Y	Y	6/8
Tropie	Y	Y	N	N	Y	Y	Y	Y	6/8
Tylmanowa	Y	Y	Y	Y	Y	Y	N	N	6/8

where: Y – criterion fulfilled; N – criterion not fulfilled.

5. CONCLUSION

Basing upon obtained numerical results, following general conclusions can be formulated:

- For all footbridges ultimate limit state is fulfilled;
- For all footbridges horizontal and vertical stiffnesses, are suitable from statical point of view and enable to fulfil statical requirements according to Polish Standard [6];
- In the case of footbridges in Rożnów and Tropie static angle of tosion of the span is very big and not fulfil requirement;
- For the footbridges in Rożnów, Tropie and Piwniczna dynamic serviceability limit state in horizontal direction is not fulfilled in accordance with [7];
- Only footbridge in Tylmanowa fulfils all assumed serviceability limit state;
- For all footbridges, no aerodynamical phenomenon (such as galloping, flutter, divergence) take place;
- Presented in this paper calculation results have been obtained taking into account only three the lowest symmetrical or quasi-symmetrical free vibration modes. In further aerodynamical calculations also the asymmetrical or quasi-asymmetrical free vibration modes should be taken into account.

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