COMPARISION OF HORIZONTAL AND TROSIONAL STIFFNESSES OF LIGHT FOOTBRIDGES UNDER WIND ACTION

G. Bosak
Cracow University of Technology, Cracow, Poland
A. Flaga
Cracow University of Technology and Lublin Technical University, Poland
T. Michałowski
Cracow University of Technology, Cracow, Poland

1. INTRODUCTION

In this article an aerodynamical analysis of few light footbridges of different measures of making span of footbridges stiff enough is presented. A model of wind load adopted in agreement with a quasi-steady concept takes into consideration not only unsteady air onflow but also motion of the structure itself. Wind load caused by vortices is neglected. In addition, it is assumed, that considered structures behave in a linear elastic way.

2. MATHEMATICAL MODEL

An analysis of footbridges is made at assumptions as follows :

- Wind action on pylons and cable systems of footbridges is of static-type;
- Wind action on spans of footbridges is considered in accordance with a quasi-steady theory;
- Linearised small vibrations of footbridges around their mean (static) position are considered;
- Vortex exciting are neglected..

Equations of quasi-steady theory enables to take into consideration of randomless of wind field and motion of span. These equations has a nonlinear differential character wiht feedbacks; this type of equations can be soluted ony on numerical way. Footbridge is treated as a structure of many finite elements, which has many degrees of freedom. For simplicity, number of degrees of freedom is reducted by using Bubnow-Galerkin method. Eigenvalues of vibrations are assumed as shape-function.

Statical and dynamical calculations are made by ALGOR.

Stochastic field of wind are generated by computer procedure, using Weighted Amplitude Wave Superposition method..

Wind characteristic is as follow:

- Powered wind profile is assumed;
- Open area;
- Average wind velocity 20.0 m/s
- Time step 0.05 s;
- Number of steps 5000 (250 s);
- Scale of turbulence 100 m;
- Three components of winfd velocity are symulated: u - on mean direction, v - on perpendicular direction, w - on verticaldirection.

Figure 1 shown example of these three components on wind velocity.

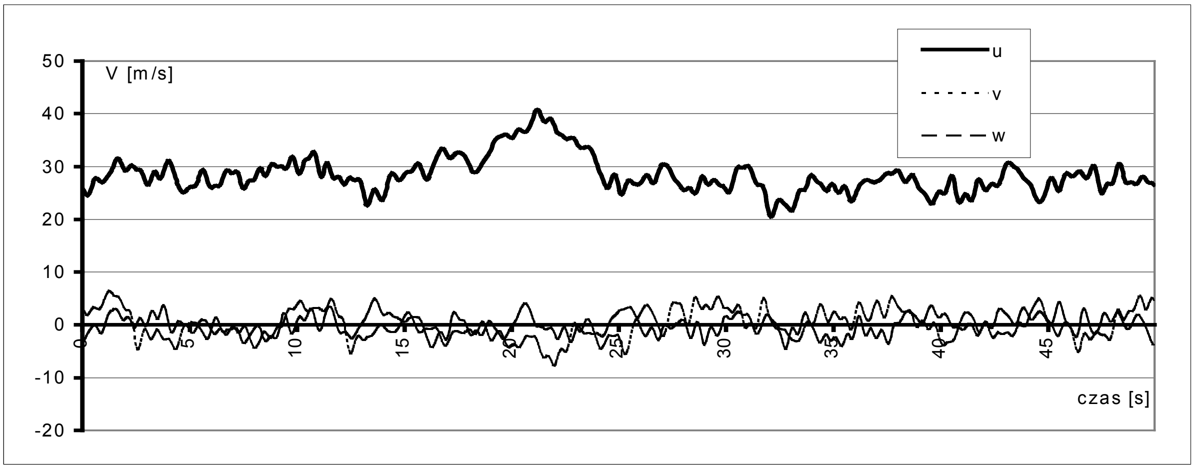


Fig.1. Example of three components of winfd velocity.

Aerodynamical coefficients assumed as follow:

Tab. 1. Aerodynamical coefficients.

Footbridge	C_x	C_y	C_m	C_{xy}	C_{yx}	C_{mm}
Without vertical trusses	0.281	0.000	0.013	0.052	4.561	0.212
With vertical trusses	2.000	0.000	0.659	0.052	4.561	0.315

2. DESCRIPTION OF THE ANALYSED FOOTBRIDGES

Few types of light footbridges are analysed in article. Theri localization is shown on fig. 2.

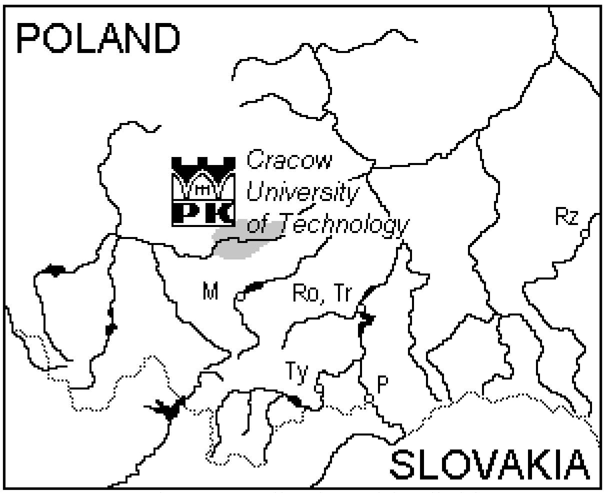


Fig. 2. Localization of foodbridges

where: M - Myślenice; P - Piwniczna; Ro - Roznów; Rz- Rzeszów; Tr - Tropie; Ty - Tylmanowa.

Among many means of making spans of light footbridges stiff enough, five different solutions are presented and analysed. First type of footbridge is a cable-ribbon suspension footbridge in Myślenice (see fig. 3). In this case transversal and torsional stiffnesses are ensured by two pairs of horizontal ropes, additionally attached to the span.

Second type of footbridge is a suspension footbridge in Piwniczna (see fig. 4), which has truss span and, moreover, two additionl horizontal steel arches.

Third type is cable-stayed foodbridge in Rzeszów (see fig. 5). In this case stiffnessess are ensured by horizontal and vertical trusses.

Fourth type of footbridge are suspension footbridge in Rożnów (see fig. 6) and in Tropie (see fig. 7). In these footbridges transversal and torsional stiffnesses are ensured mainly by inclined cable system.

Fifth type of footbridge is a cable-stayed footbridge (see fig. 8) in Tylmanowa.(namet Tylmaniwa III) Horizontal stiffness of this footbridge is mainly ensured by horizontal truss, what is the most often case encountered in practice. Torsional stiffness comes from two vertical trusses and stays. Moreover, horizontal stiffness is amplyfied in adition by steel ortothropic plate.

Sixsth type is another footridge (see fig. 9) in Tylmanowa (Tylmanowa II). Horizontal stiffness is ensured by horizontal truss, vertical stiffness is ensured only by tensions.

Moreover, cable-stayed footbridge with cable-bar elements are analysed (see fig.10). In this case both horizontal and torsional stffnesses come from horizontal and vertical cable-bar systems mainly. This footbridges, named K-1 and K-5, are presented in fig. 10 and 11.

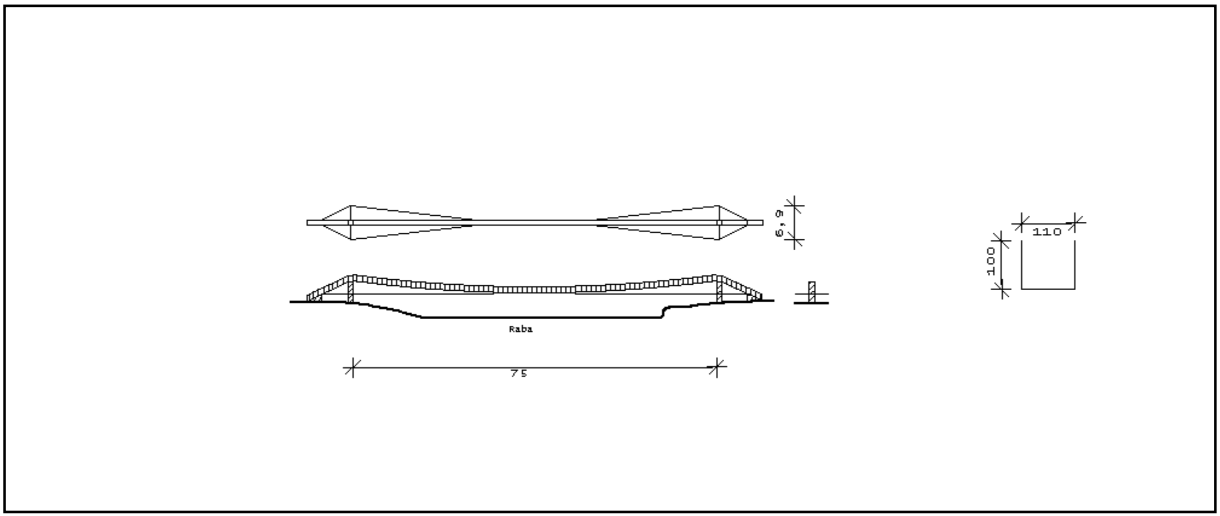


Fig. 3. Footbridge in Myślenice

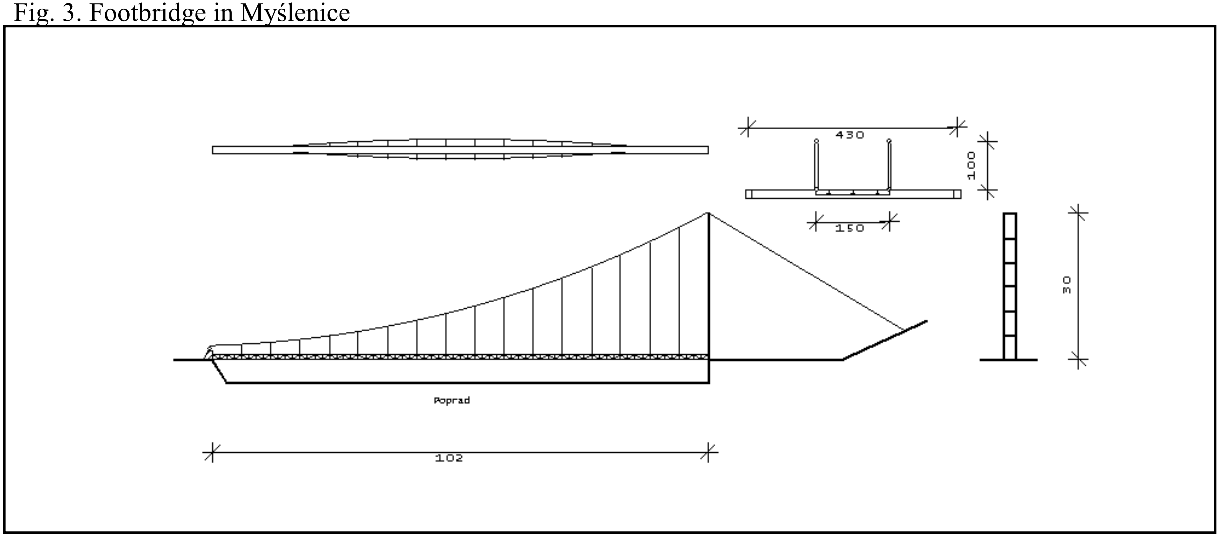


Fig. 4. Footbridge in Piwniczna

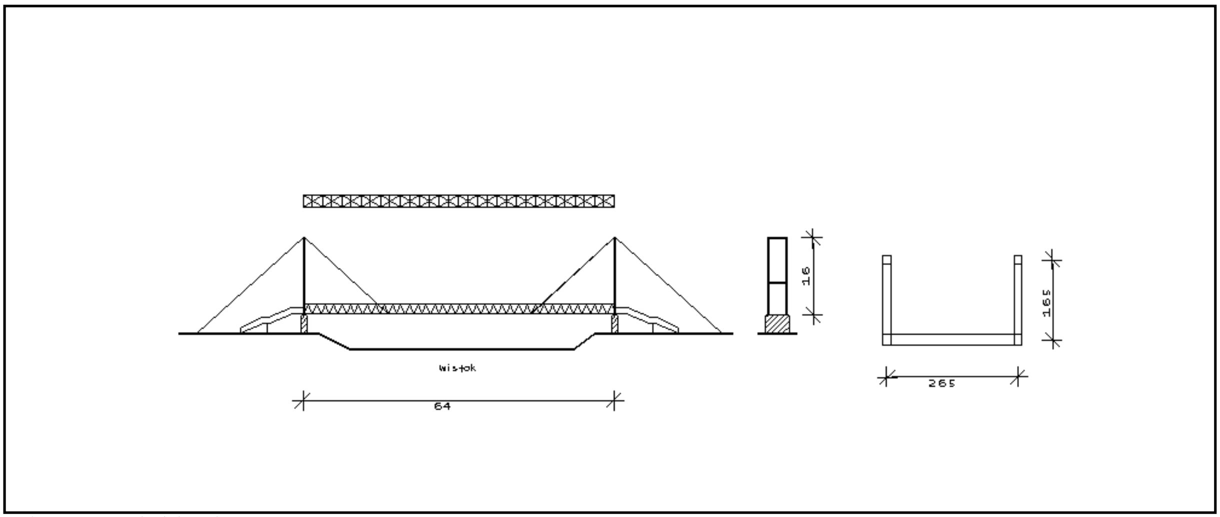


Fig. 5. Footbridge in Rzeszów

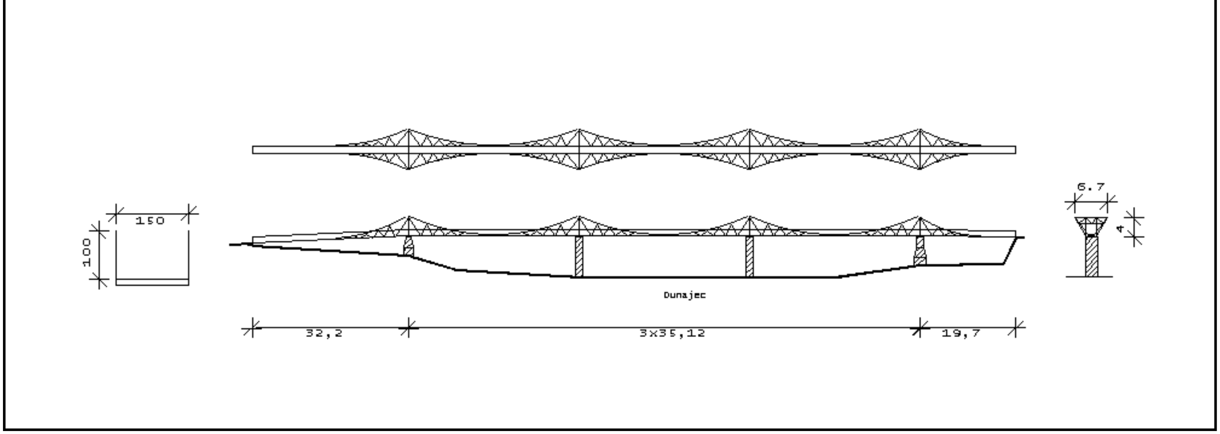


Fig. 6. Footbridge in Rożnów

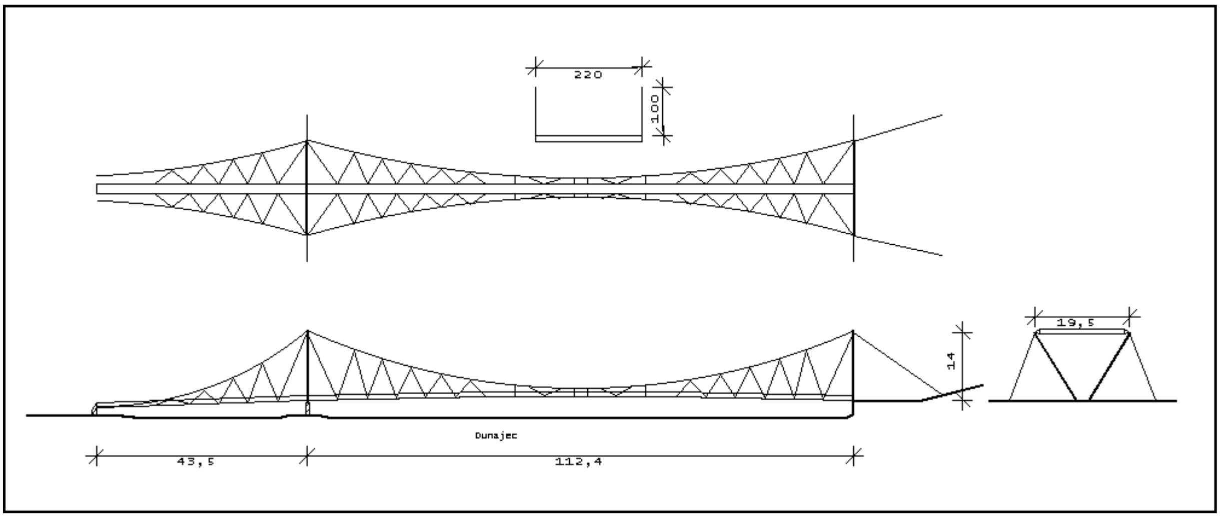


Fig. 7. Footbridge in Tropie

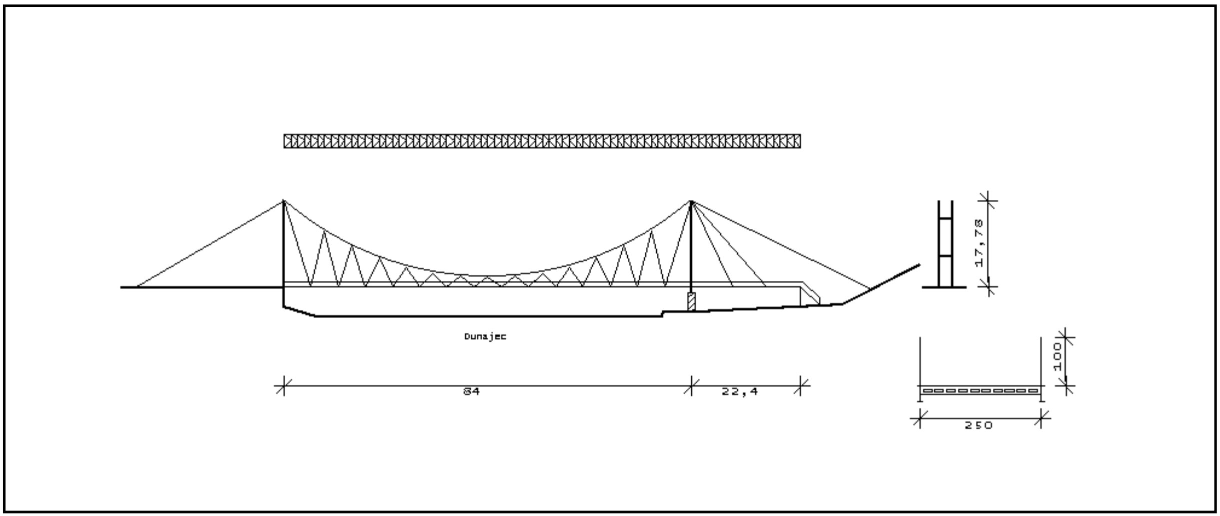


Fig. 8. Footbridge in Tylmanowa

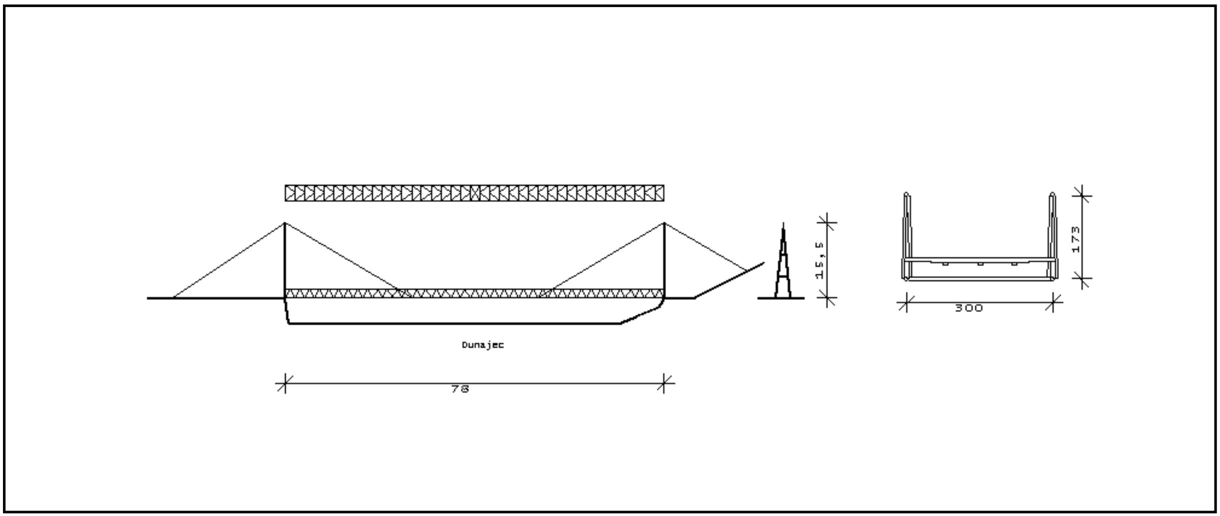


Fig. 9. Footbridge in Tylmanowa (pieszo-jezdna)

.

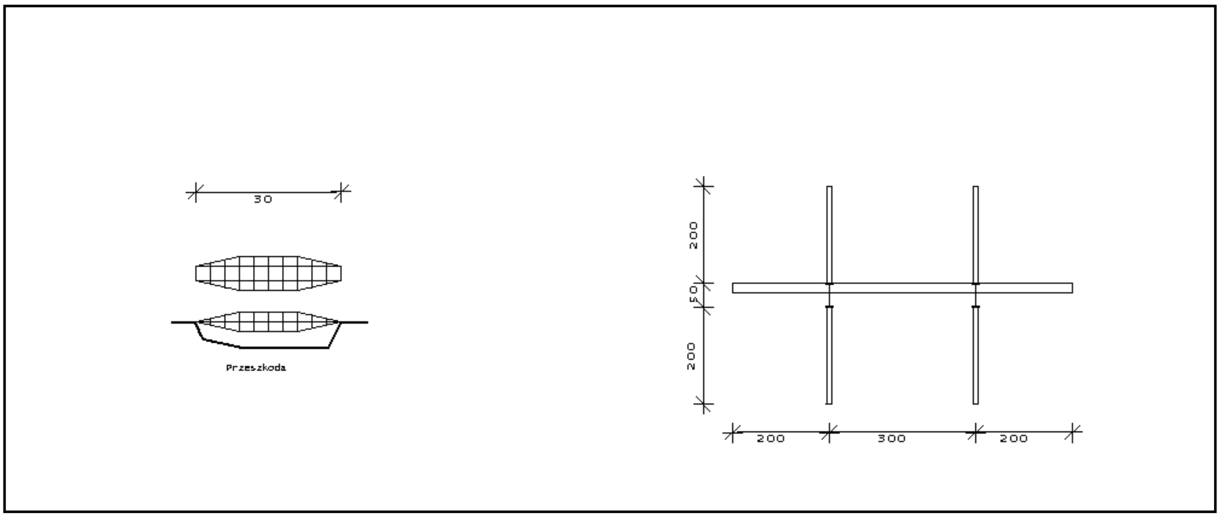


Fig. 10. Cable-bar footbridge, one modulus (K-1)

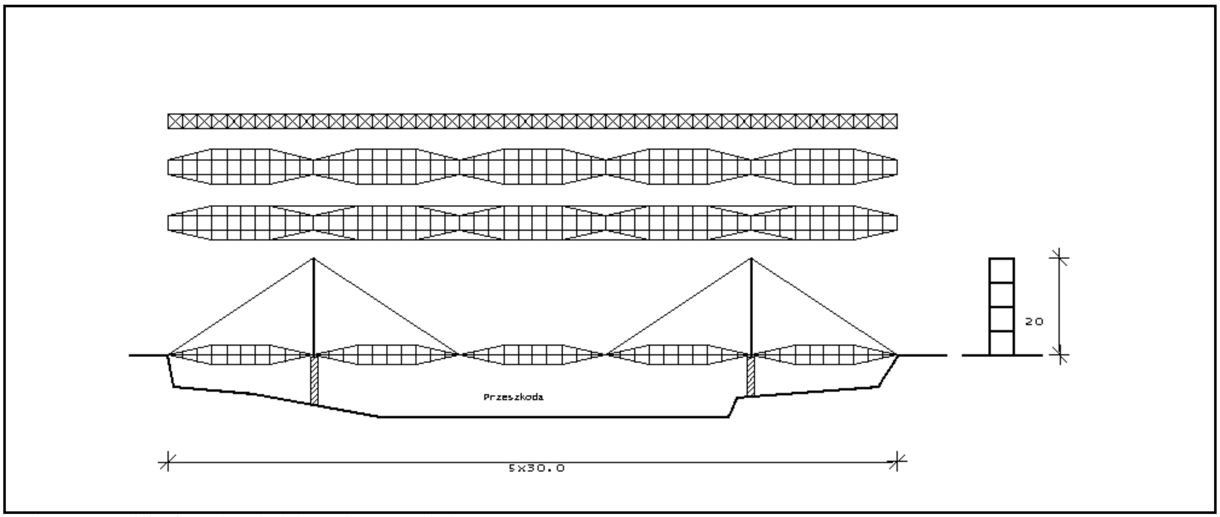


Fig. 11. Cable-bar footbridge, five modulus (K-5)

3. STATICAL CALCULATIONS

In statical calculations two cases are take into consideration; gravity load and gravity load with wind load. Difference of displacemens between these two cases are treated as displacement against wind load.

In analysis three nonlinear parameter are used::

$$\psi = y / L \tag{1}$$

$$\zeta = z / L \tag{2}$$

where: y, z – horizontalor vertical displacement of span; L - length of span.

$$\gamma = m / q \tag{3}$$

where: m – death-weight; q – imposed load.

These parameters for analysed footbridges are put together in tab. 2.

Tab. 2. Statical displacements of footbridges.

Kładka	γ	Staticaldisplacements		
		1000 ψ	1000 ξ	ϕ [rad]
Footbridges				
Tropie	0.101	1.010	0.353	0.059
Rożnów	0.165	2.414	0.367	0.070
Myślenice	0.198	0.529	0.033	-0.056
Tylmanowa II	0.312	0.047	0.000	0.000
Piwniczna	0.372	1.031	0.010	0.009
Rzeszów	0.438	0.139	-0.047	0.009
Kładki pieszo-jezdne				
K-1	0.512	0.297	0.000	0.000
K-5k	0.512	0.033	0.189	0.000
K-5d1	0.512	2.256	0.222	0.004
K-5d2	0.512	1.622	0.211	0.004
Tylmanowa III	1.064	0.223	0.000	0.000

All of these footbridges can be divided into two groups: with inclined cable system (for example Tropie and Rożnów) and without inclined cable system (for example Piwniczna; the same shape for all other footbridges). Comparison between the static horizontal aerodynamics force F and the horizontal displacement y of span of these footbridges are shown in figs 12 and 13.

Moreover, the trajectories of plan motion of center points of spans permit to divide footbridges in the same way. Fig 14 shown trajectory for footbridge with inclined cable system (Rożnów); fig. 15 for footbridge without inclined cable system (Tylmanowa II)

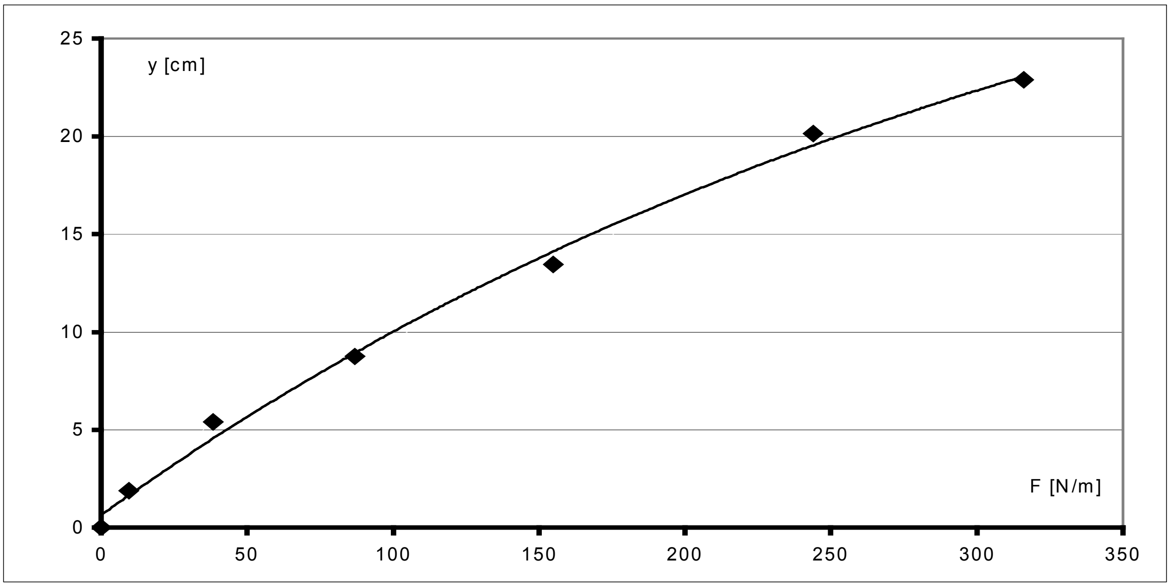
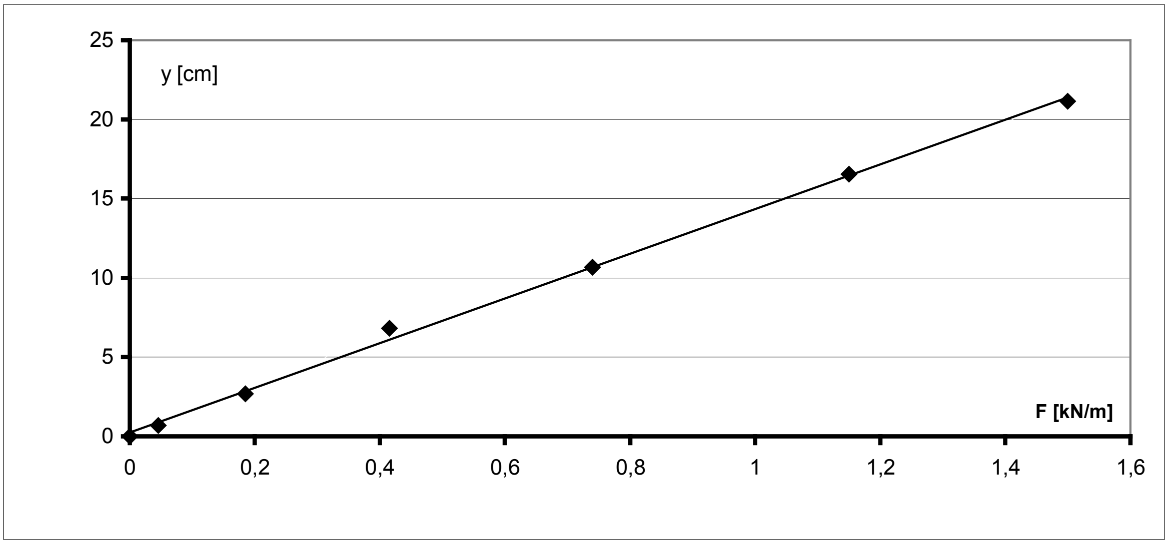


Fig. 12. Dependence between the static horizontal aerodynamic force F and the horizontal displacement y of the Rożnów footbridge span



Rys. 13. Dependence between the static horizontal aerodynamic force F and the horizontal displacement y of the Tylmanowa II

Footbridges with inclined cable system behave as pendulum. Stiffenes of span, making by cables cables neglected; stiffenes of footbridge comes from mainly inclined cable system. Fot other types of structure, footbridges behawe as beam.

4. DYNAMICAL CALCULATIONS

Eigenvalues and eigenfrequences of free vibration of footbridges are calculated. Tab. 3. shown characteristics of eigenvalues andeigenfrequencies.

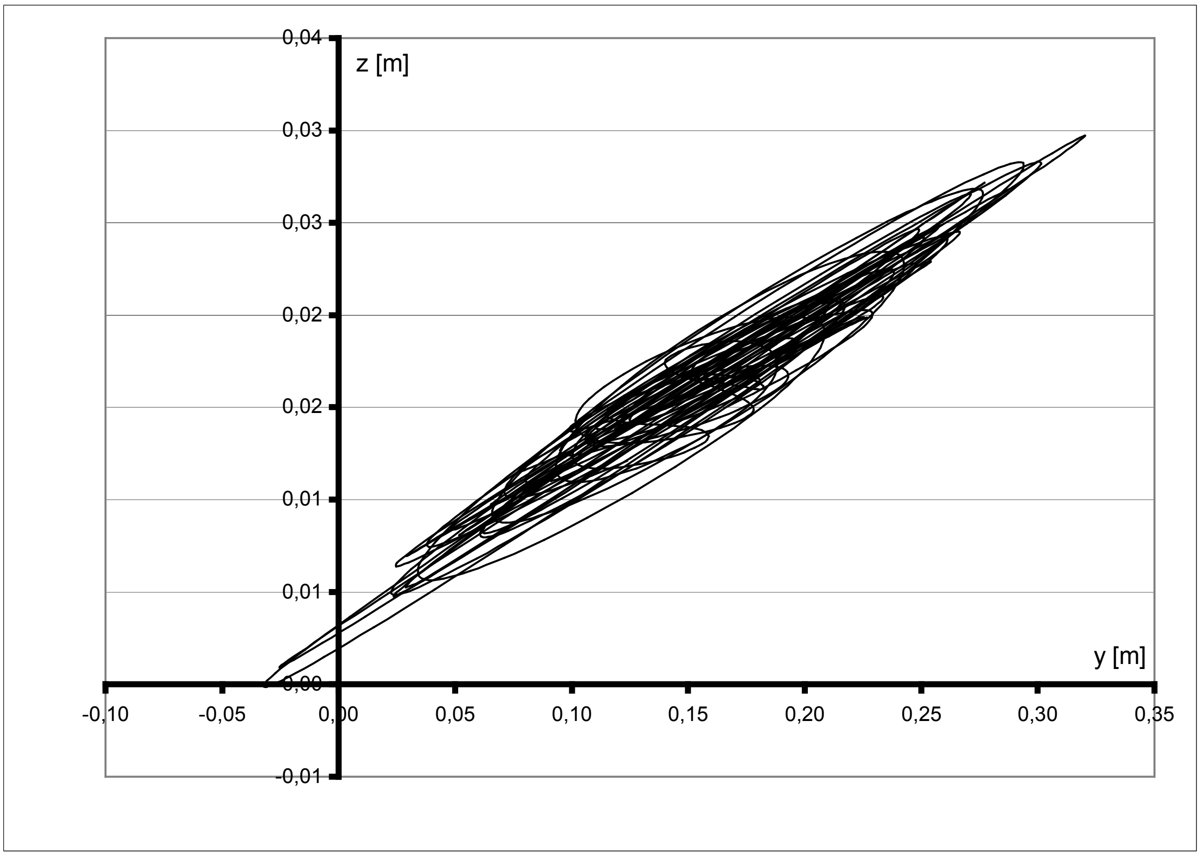


Fig. 14. Planar motion of the central point of span of the Rożnów footbridge.

Fig. 15. Planar motion of the central point of span of the Tylmanowa footbridge.

Tab. 3. Eigenvalues and eigenfrequencies						
Footbridge	[Hz]					
Myślenice	H S 0.963	H V T A 1.096	V T S 1.202	V T S 1.223		
Piwniczna	H T S 0.588	H T S 0.685	H T A 0.936	V S 1.156	H T A 1.326	V A 1.404
Rożnów	H T A 0.610	H T A 0.638	H T S 0.692	V S 1.006		
Rzeszów	H T S 1.778	V S 2.055	H T S 3.609	H T A 3.126		
Tropie	H T S 0.500	H T V S 0.813	H T A 0.871	H T A 1.223		
Tylmanowa II	H S 1.215	1.803 H S	V S 1.862	T S 1.959		
K-1	T S 1.595	V S 2.274	V T A 2.352	H T S 2.647	V A 3.006	
K-5	H S 0.762	H T S 0.893	V S 1.061	T A 1.077	V A 1.238	
Tylmanowa III	H S 1.080	H A V S 1.469	H A 1.658	H S 2.144		

where: H - horizontal vibrtions; V - vertival vibrations; T - torsional vibrations; S - symmetrical vibrations, A - assymetrical vibrations.

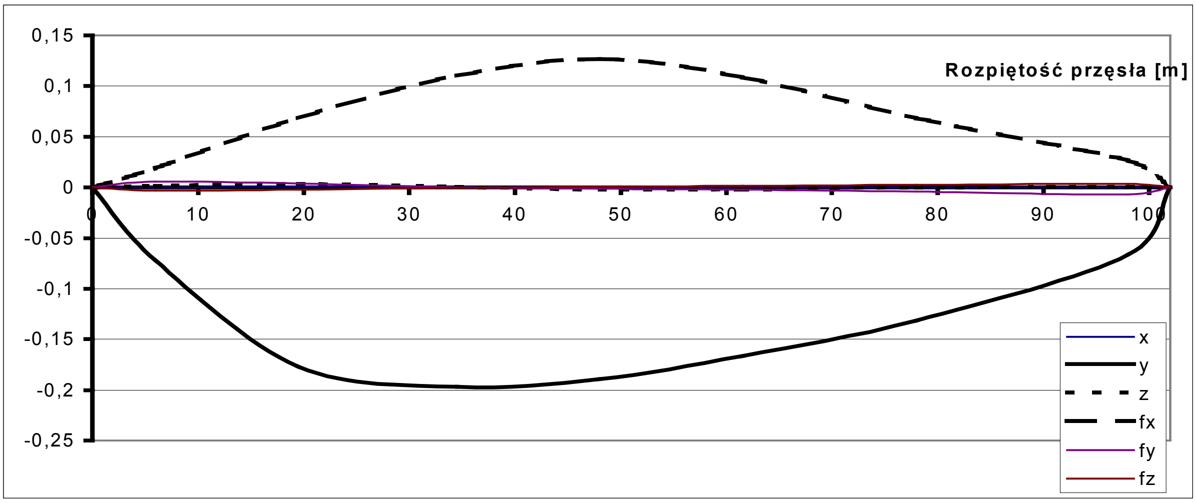


Fig 16. Spatial character of vibration modes of the suspension footbridges in Piwniczna (HTS, f=0.588 Hz)

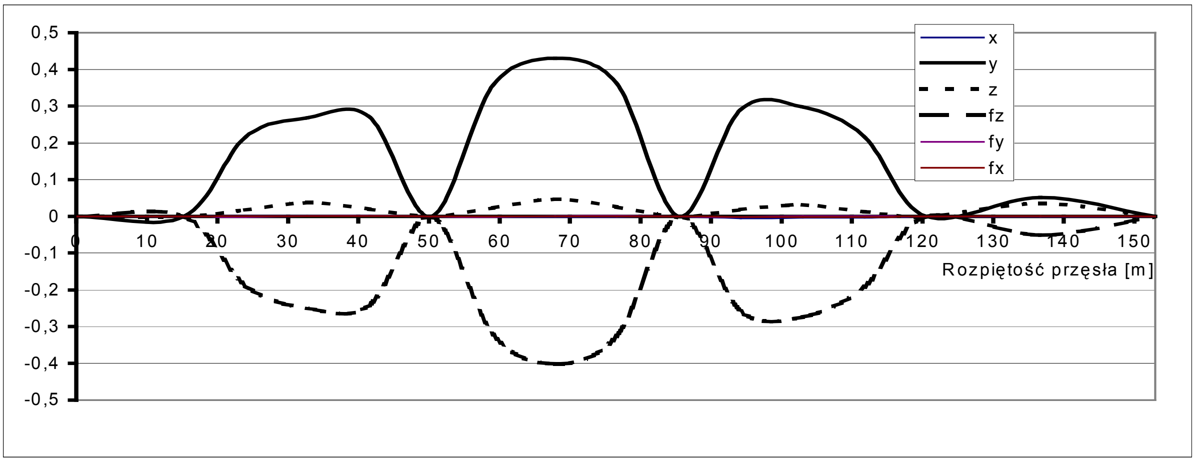


Fig 17. Spatial character of vibration modes of the suspension footbridges in Rożnów (HTA, f=0.610 Hz)

In calculations, deformated construction is considered. Because of this, vibrationmodes are not so clear; have spatial character.

Tab. 4. shows amplitudes of vibrations for considered footbridges.

Tab. 4. Amplitudes of vibration

Kładka	γ	Dynamical displacement		
		$1000\ \psi$	$1000\ \xi$	ϕ [rad]
Footbridges				
Tropie	0.101	0.063	0.009	0.004
Rożnów	0.165	0.861	0.071	0.030
Myślenice	0.198	0.192	0.502	0.043
Tylmanowa II	0.312	0.018	0.066	0.000
Piwniczna	0.372	0.345	0.142	0.035
Rzeszów	0.438	0.016	0.039	0.000
Kładki pieszo-jezdne				
K-1	0.512	0.033	0.067	0.001
K-5d2	0.512	0.556	0.178	0.004
Tylmanowa III	1.064	0.122	0.038	0.001

6. SERVICEABILITY LIMIT STATE

To evaluate of the serviceability limit state of the analysed footbridges, following standards and proposal have been taken into account:

- Polish Standard

According to the Polish Standard values of vertical statical displacement of span and aperionof horziontal vibrations should be as follows:

$$\psi_S < 0.0026 \qquad (4)$$

$$T_H < 0.1L \qquad (5)$$

where L -length of the span [m]; T_H - period of the first natural horizontal vibrations [s].

- Authors's proposal with respect to angle of torsion of span:

In winter months span of the footbridges can be ice-coated. Because of this, angle of the torsion of the span can't be too big; so our proposition is as follow:

$$\phi < 3^o = 0.052\ rad \qquad (6)$$

This condition can be compared with (4). Average length of span for considered footbridges is equal 80m, average widht of span is equal 2 m. According to (4), acceptable vertical displacement is equal 20.8 cm. According to (6), difference between edge of span is equal 10.8cm., nearly half of acceptable vertical displacement.

- Flaga's proposals.

These proposals in form of dependencies between rms of acceleration of span and frequencies of vibrations are presented on figs 18, 19 and 20, together with numerical results, obtained for the analysed footbridges (1 - Piwniczna, 2 - Rożnów, 3 - Tropie, 4 - Tylmanowa). Acceptable values lie below solid lines.

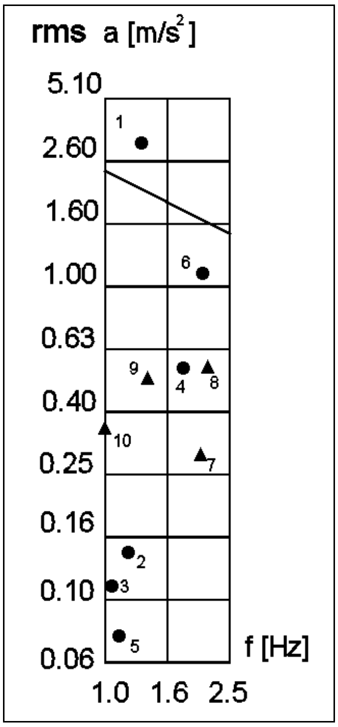


Fig. 18. Acceptable curves and numerical results for vertical vibrations

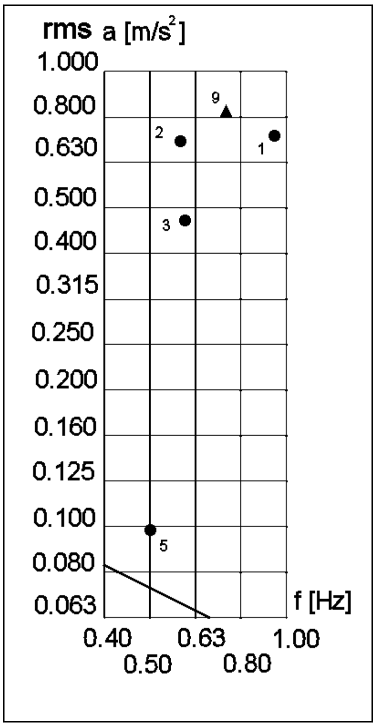


Fig. 19. Acceptable curves and numerical results for horizontal vibrations

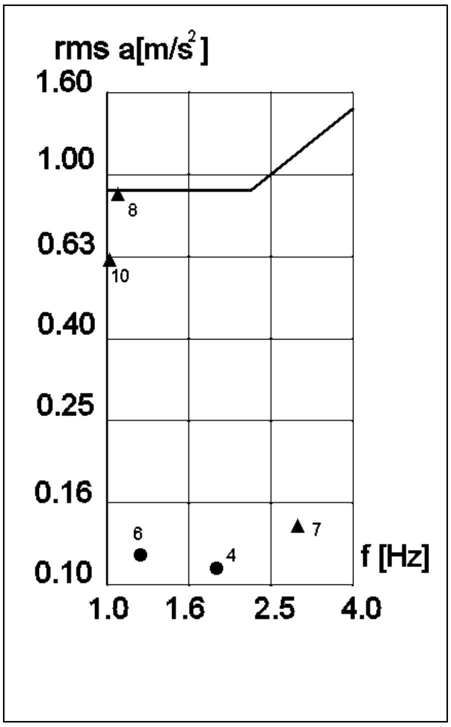


Fig. 20. Acceptable curves and numerical results for horizontal vibrations

1 - Myślenice, 2 - Piwniczna, 3 - Roznów, 4 - Rzeszów, 5 - Tropie, 6 - Tylmanowa II, 7 - K-1, 8 - K-2, 9 - K-5, 10 - Tylmanowa III.

- Moreover, another requirements with respect to not allowable frequency bands for footbridges are recommended byauthors.

For footbridges vibrations caused by pedestrain excitation can be dangerous for the following horizontal, vertical and torsional frequencies bands:

$$f_V \notin (1.65 \div 2.35)\ Hz \tag{7}$$

$$f_H \notin (1.9 \div 2.1)\ Hz \tag{8}$$

$$f_T \notin (1.65 \div 2.35)\ Hz \tag{9}$$

where: f_V, f_H, f_T – the lovest frequencies of vertical,horizontal and torsional vibrations.

Results of analysis of serviceability limit state for considered footbridges are presented in table 5.

Tab. 5. Spełnienie warunków użytkowalności

Footbridge	Statical displacements			Amplitudes		Excitation by pedestrain		
	y	Z	ϕ	y	Z	Y	z	ϕ
Footbridge								
Myślenice	Y	Y	N	N	N	Y	Y	Y
Piwniczna	Y	Y	Y	N	Y	Y	Y	Y
Rożnów	Y	Y	N	N	Y	Y	Y	Y
Rzeszów	Y	Y	Y	Y	Y	Y	N	N
Tropie	Y	Y	N	N	Y	Y	Y	Y
Tylmanowa II	Y	Y	Y	Y	Y	Y	N	N
Kładki pieszo-jezdne								
K-1	Y	Y	Y	Y	Y	Y	Y	Y
K-5d2	Y	Y	Y	N	Y	Y	Y	Y
Tylmanowa III	Y	Y	Y	Y	Y	Y	Y	N

where Y – criterion fulfilled; N – criterion not fulfilled.

7. CONCLUSION

Basing upon obtained numerical results, following general conclusions can be formulated:

- For all footbridges ultimate limit state is fulfilled;
- For all footbridges horizontal and vertical stiffnesses, are suitable from statical point of view and enable to fulfil statical requirements according to Polish Standard;
- In the case of footbridges in Rożnów and Tropie static angle of tosion of the span is very big and not fulfil require;
- For the footbridges in Rożnów, Tropie and Piwniczna dynamic serviceability limit state in horizontal direction is not fulfiled in accordance with;
- Only footbridge in Tylmanowa fullfills all assumed serviceability limit state;
- For all footbridges, no aerodynamical phenomenon (such as galloping, flutter, divergence) take place;
- Presented in this paper calculation results have been obtained taking into account only three the lowest symmetrical or quasi-symmetrical free vibration modes. In further aerodynamical calculations also the asymmetrical or quasi-asymmetrical free vibration modes should be taken into account.

- Lekkie kładki dla pieszych (Myślenice, Rożnów, Tropie) wypadają bardzo niekorzystnie w porównaniu z innymi rodzajami kładek. Kładka wstęgowa nie spełnia żadnego z warunków dynamicznych i warunku statycznego kąta skręcenia; kładki o pochylonym układzie lin mają kłopoty ze spełnieniem zarówno warunków statycznych, jak i dynamicznych;
- Trajektorie ruchu kładek o pochyłym układzie lin są podobne do ruchu wahadła, ich sztywność pionowa, pozioma i skrętna pochodzi tylko od systemu nośnego, sztywność przęsła jest pomijalnie mała;
- W przypadku kładki w Myślenicach stwierdzono możliwość wystąpienia niestabilności aerodynamicznej;
- Spośród kładek dla pieszych najoptymalniejszym rozwiązaniem wydaje się przęsło o poziomym stężeniu kratownicowym (Rzeszów, Tylmanowa II). Jedynie te przęsła spełniają wszystkie warunki statyczne i dynamiczne;
- Kładki pieszo-jezdne, wykonane z elementów prętowo-cięgnowych, są konkurencyjne w stosunku do innych rozwiązań konstrukcyjnych kładek pieszo-jezdnych, wyraźnie zaś lepsze od lekkich kładek dla pieszych;
- Stężenia poziome w postaci łuków stalowych lub cięgnowych (Piwniczna, kładki studialne) są gorszym rozwiązaniem od poziomego stężenia w postaci kratownic stalowych lub cięgnowych;
- Studialne kładki prętowo-cięgnowe, kładka w Piwnicznej oraz lekkie kładki (Myslenice, Rożnów, Tropie) lepiej radzą sobie ze spełnieniem warunków, wynikających ze wzbudzenia kładki przez tłum pieszy, niż kładki o poziomych stężeniach kratownicowych;

Moduły prętowo-cięgnowe powinny być analizowane przy użyciu nieliniowej statyki, z uwagi na silnie nieliniową pracę cięgien. Okazało się jednak, że jako całość zachowują się jak konstrukcja liniowa, co widoczne jest przy analizie zależności między przemieszczeniami pionowymi a wielokrotnością obciążenia tłumem oraz przy analizie zależności między przemieszczeniem poziomym a poziomą siłą aerodynamiczną.

[1] Flaga A.; Quasi-steady theory in aerodynamics of slender structures; Sonderforschungsbereich 151 - Tragwerksdynamik, Wissenschaftliche Mitteinlungen; Berichte Nr. 25. Bochum, 1994, s.135.
[2] PN – 82 / S-10052 Obiekty mostowe, konstrukcje stalowe, projektowanie
[3] Flaga A.; Problemy oceny wpływu drgań na ludzi znajdujących się na mostach; Inżynieria i Budownictwo 4/2002, 182÷187
[4] Vibration problems in structures; practical guidelines; Bulletin dinformation No 209 CEB, praca zbiorowa pod redakcją H. Bachmana,Lozanna 1991