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Study of aerodynamical behaviour of suspension footbridges against wind load

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Abstract:
An aerodynamical analysis of four existing light footbridges of different measures of making span of footbridges stiff enough (i.e. inclined cable system, horizontal truss, horizontal arches) is presented. A model of wind load adopted in agreement with a quasi-steady concept takes into consideration not only unsteady air onflow but also motion of the structure itself. Wind load caused by vortices is neglected. In addition, it is assumed, that considered structures behave in a linear elastic way.

Theoretical approach to a problem

An analysis of four existing light footbridges is made at assumptions as follows :

- Wind action on pylons and cable systems of footbridges is of static-type;
- Wind action on spans of footbridges is considered in accordance with a quasi-steady theory [1, 5].
- Linearised small vibrations of footbridges around their mean (static) position are considered.

Quasi-steady theory enables to consider several aerodynamic phenomena, such as galloping, special type of flutter, buffeting, and divergence. Three components of wind load include static and dynamic wind actions on span footbridges. These three components of wind load, i.e. drag force, lift force and aerodynamic moment, can be given by :

$$q = \rho(\bar{u} + u'_s)^2 / 2 \tag{1}$$

$$W = (v'_s + \xi^* \sin\bar{\alpha} - \eta^* \cos\bar{\alpha})/(\bar{u} + u'_s) - [X_G \cos(\bar{\alpha} - \varepsilon) + Y_G \sin(\bar{\alpha} - \varepsilon)]/(\bar{u} + u'_s)]\varepsilon^* - \varepsilon \tag{2}$$

$$w_x = q\,H\,[C_x + C_{xy}\,W] \tag{3}$$

$$w_y = q\,H\,[C_y + C_{yx}\,W] \tag{4}$$

$$w_m = q\,H^2\,[C_m + C_{mm}\,W] \tag{5}$$

where: ρ - density of air; *u, u', v'*, α, η, ξ, ε – as in fig. 1; *H* – characteristic diameter of cross-section; X_G and Y_G – coordinate of geometric centre of the outline curve of cros-ssection; C_x , C_{xy} , C_y , C_{yx} , C_m, C_{mm} – aerodynamic coe-fficients; $\bar{\square}$ – time averaged value; $\square^* = d\square/dt$; $\square'$ – fluctuations; $\square_s$ – spatially averaged value.

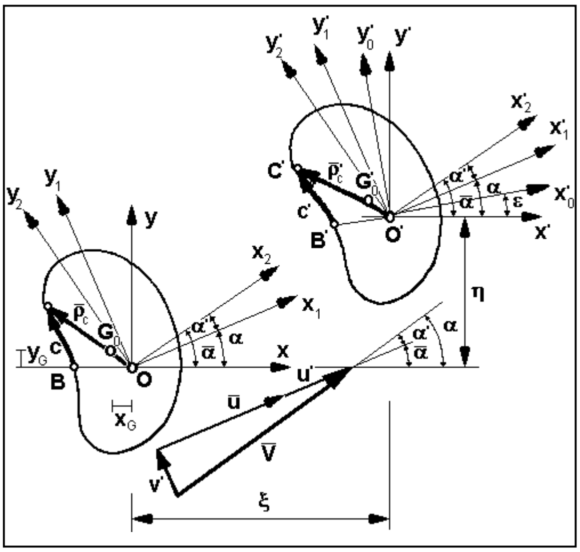

Fig. 1 Systems of coordinates and relations between different geometrical quantities in the case of a moving structure and an unsteady air onflow

Mathematical model of the htree degrees of freedom system as a substitutional system of existing structure

Spans of the considered footbridges should be treated as a slender structures, which can be divided into m elements. Each of them is represented by the inner point *k* of the cross-section (see fig. 2).

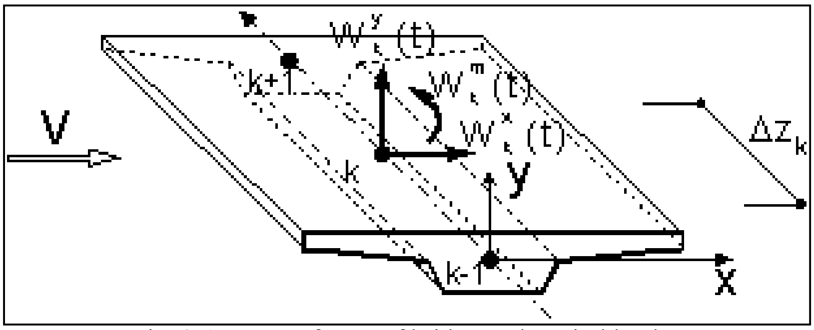

Fig. 2 A sector of span of bridge under wind load

Formulas (1-5), which are related to all points of span next are transformed to the global system of coordinates. It is assumed, that the structures behave in a linear elastic way. Their motion can be described by a matrix equation :

$$[M]\{r^{**}\} + [C]\{r^{*}\} + [K]\{r\}=\{w\} \qquad (6)$$

The Bubnow-Galerkin's method is adopted. As a result, a system of many degrees of freedom is substituted by a representative system of three degrees of freedom. Motion of the substitutional system under wind load described by the formulas (1-6) can be expressed by :

$$M_j\Psi_j^{**}(t) + C_j\Psi_j^{*}(t) + K_j\Psi_j(t) = \qquad (7)$$

$$= \sum_k^m \langle X^j\{A(t)\}_k^{\xi T} + Y^j\{A(t)\}_k^{\eta T} + \Phi^j\{A(t)\}_k^{\varepsilon T}\rangle[\chi(t)]_k$$

$$\{A(t)\}_j^{\xi T} = [C_xW_K(t),\ C_{xy}W_K(t)a_k(t),\ C_{xy}W_K(t)b_k(t),\ -C_{xy}W_K(t)c_k(t),\ -C_{xy}W_K(t)d_k(t),\ C_{xy}W_K(t)e_k(t),\ -C_{xy}W_K(t)] \qquad (8)$$

$$\{A(t)\}_j^{\xi T} = [C_yW_K(t),\ C_{yx}W_k(t)a_k(t),\ C_{yx}W_k(t)b_k(t),\ -C_{yx}W_k(t)c_k(t),\ -C_{yx}W_k(t)d_k(t),\ C_{yx}W_k(t)e_k(t),\ -C_{yx}W_k(t),] \qquad (9)$$

$$\{A(t)\}_j^{\varepsilon T} = [HC_mW_K(t),\ HC_{mm}W_K(t)a_k(t),\ HC_{mm}W_K(t)b_k(t),\ -HC_{mm}W_K(t)c_k(t),\ -HC_{mm}W_K(t)d_k(t),\ HC_{mm}W_K(t)e_k(t),\ -HC_{mm}W_K(t)] \qquad (10)$$

$$\{\chi(t)\}^T_k=[1,\ 1,\ \xi^*(t)_k,\ \eta^*(t)_k,\ \varepsilon^*(t)_k,\ \varepsilon^*(t)_k\varepsilon(t)_k,\ \varepsilon(t)_k,] \quad (11)$$

$$W_k(t) = H\rho u^2\Delta z_k \quad (12)$$

$$u= u+ u' \quad (13)$$

$$a_k(t,z) = v'(t,z) / u(t,z) \quad (14)$$

$$b_k(t,z) = sin\alpha(z) / u(t,z) \quad (15)$$

$$c_k(t,z) = cos\alpha(z) / u(t,z) \quad (16)$$

$$d_k(t,z) = [X_G\ \ cos\alpha(z) + Y_G\, sin\alpha(z)] / u(t,z) \quad (17)$$

$$e_k(t,z) = [Y_G\ \ sin\alpha(z) - Y_G\, cos\alpha(z)] / u(t,z) \quad (18)$$

where : $\Psi_j(t)$ – j-th generalized coordinate; X^j, Y^j, Φ^j – translation and rotation coordinates of j-th free vibration mode of the point which represents k-th structural element; m- number of structural elements; M_j, C_j K_j – generalized mass, damping and stiffeness, Δz_k – length of k-th structural element.

The complex form of the non-linear set of differential equations is resolved in a numerical way. The Newmark method of integration of the motion equations is adopted. The algorithm of the Newmark method is based on the own computer programme which gives the general displacements ξ, η, ε for each time step in k representative points for particular structural elements.

Description of the analysed structures

Among many means of making spans of light footbridges stiff enough, three different solutions are presented and analysed. First type of footbridge are suspension footbridges in Rożnów (see fig. 3) and in Tropie (see fig. 4). In these footbridges transversal and torsional stiffnesses are ensured mainly by inclined cable system.

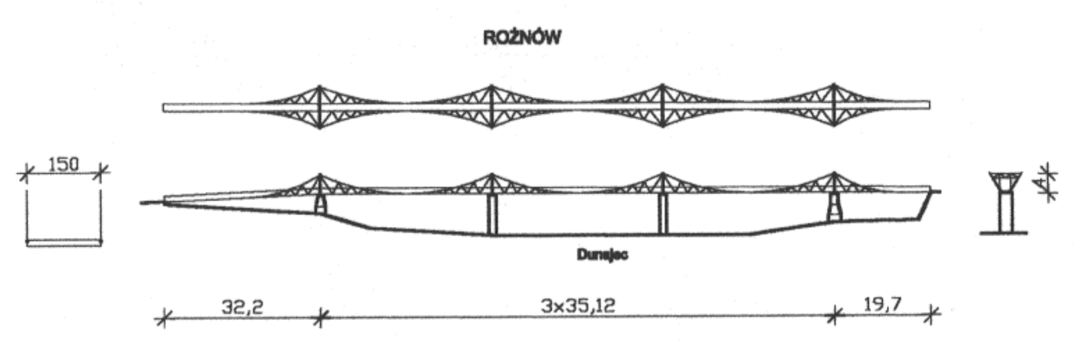

Fig. 3. Footbridge in Roznów

Second type of footbridge is a suspension footbridge in Tylmanowa (see fig. 5). Horizontal stiffness of this footbridge is mainly ensured by horizontal truss, what is the most often case encountered in practice. Torsional and vertical stiffness come from suspension cables and hangers.

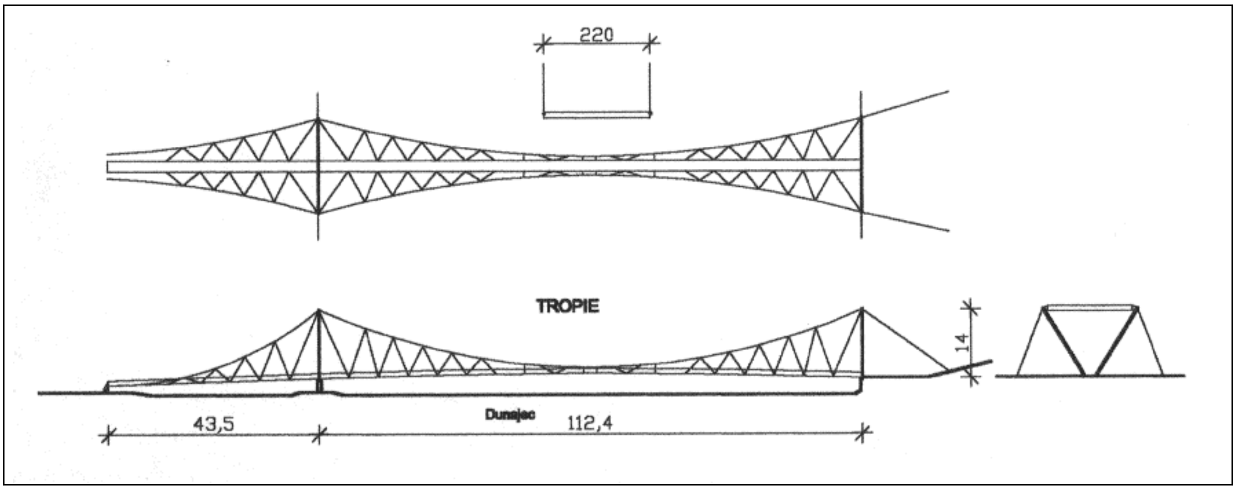

Fig. 4. Footbridge in Tropie

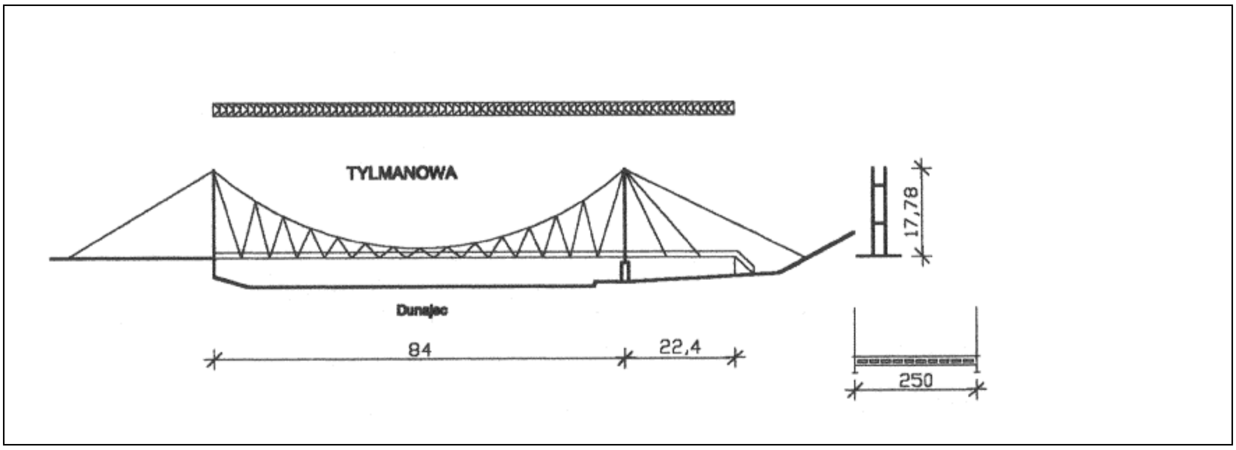

Fig. 5. Footbridge in Tylmanowa

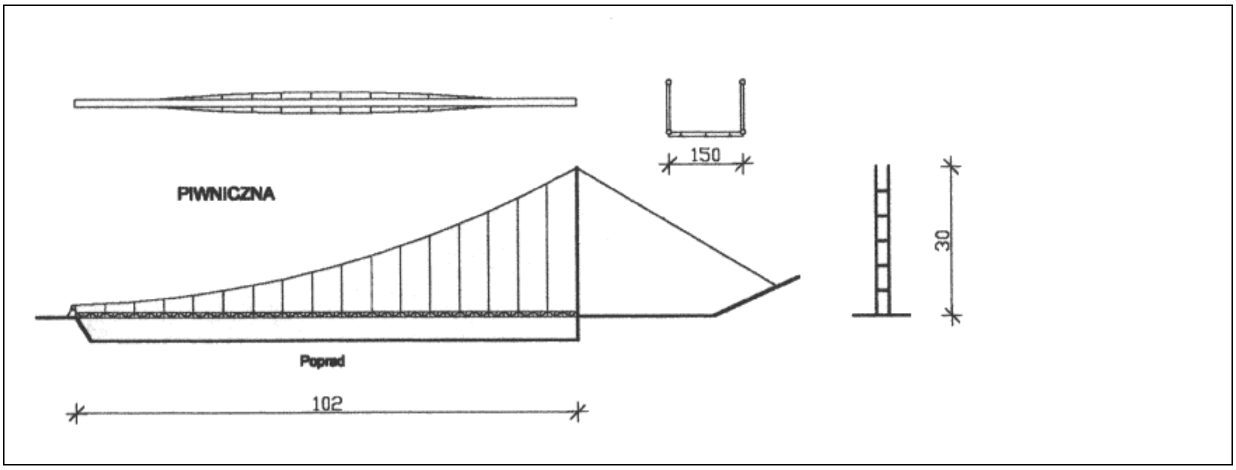

Fig. 6. Footbridge in Piwniczna

Third type of footbridge is a suspension footbridge in Piwniczna (see fig. 6), which has vertical truss and two additional horizontal steel arches. Moreover, horizontal stiffness is amplyfied in adition by steel ortothropic plate.

Exemplary calculation results

General assumptions

The following general assumptions have been assumed in numerical calculations:

- Linearised small vibrations of footbridges, around their static position, are considered. This means, that vibrations of footbridges influenced by static component of wind load, come in question. Because of this, a character of vibration modes can be complicated and not so „clear" (vertical, horizontal etc.), but of a spatial character (quasi-vertical, quasi-symmetrical, etc.).
- With respect to horizontal, vertical and torsional directions, the lowest three symmetrical or quasi-symmetrical vibration modes, showed in tab. 1, are considered (see example in fig. 7).

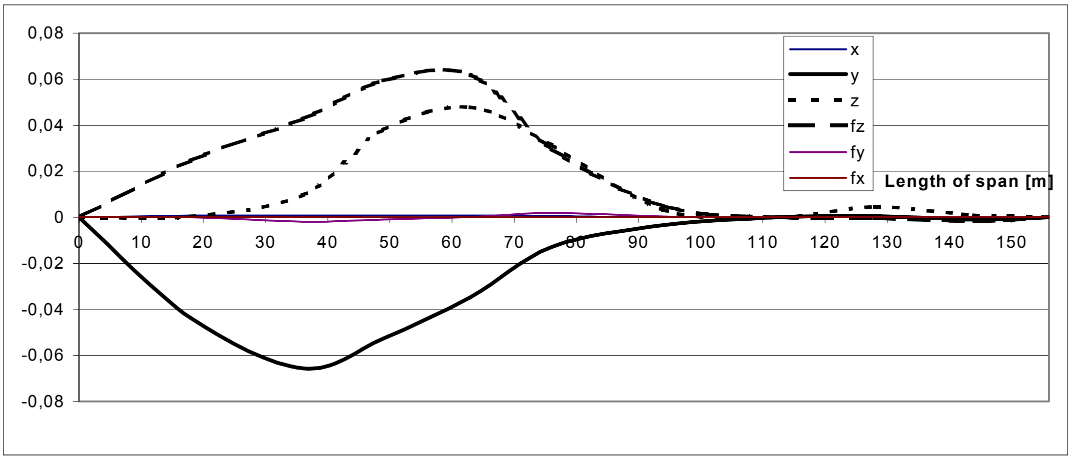

Fig. 7. Spatial character of vibration modes of the suspension footbridges in Tropie (HTVS, f=0.813 Hz)

Table 1. Free vibration modes and eigenfrequencies of the footbridges						
Footbridge	Frequencies [Hz] and modes					
Roznów	H T A 0.610	H T A 0.638	H T S 0.692	V S 1.006		
Tropie	H T S 0.500	H T V S 0.813	H T A 0.871	H T A 1.223		
Tylmanowa	H S 1.215	1.803 H S	V S 1.862	T S 1.959		
Piwniczna	H T S 0.588	H T S 0.685	H T A 0.936	V S 1.156	H T A 1.326	V A 1.404
S – symmetrical or quasi-symmetrical mode, A – antissymmetrical or quasi-antissymmetrical mode, H – horizontal or quasi-horizontal mode, V – vertical or quasi-vertical mode, T – torsional or quasi-torsional mode, Classification of the vibration modes of the footbridges is rather complicated						

Data assumed in calculations

- Wind characteristics :

- wind is simulated by WAWS method;
- von Kármán spectrums of three components of wind are adopted (k = 0.002, z_o = 10 m);
- scale of turbulence is equal to 100 m;
- power wind profile is assumed (α = 0.24);
- mean wind velocity 10 m above ground $\underline{u}$ = 20 m/s.

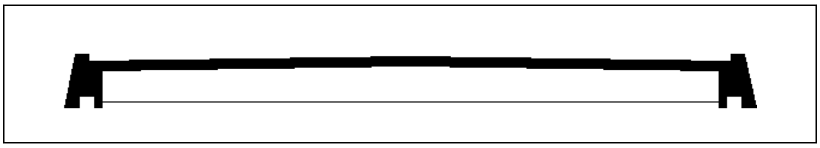

Fig. 8. Common assumed shape of the considered span cross-section of footbridges

- Aerodynamical coefficients are assumed as for a cross-section of span showed in fig. 8, so the aerodynamical coefficients are as follows :
  C_x = 0.214,
  C_{xy} = 0,
  C_y = -0.067,
  C_{yx} = 7.448,
  C_m = 0.012,
  C_{mm} = 0.206.
- Logarythmic decrement of damping Δ = 0.040.

Response of the structures

For the each footbridge mean, maximum and r.m.s. value of horizontal, vertical and torsional displacements have been obtained. In analysis of aerodynamic behaviour of footbridges, several non-dimensional parametrs have been used:

- Ratio of the static (average; ψ_S), and amplitude, (dynamic; ψ_A) horizontal displacement to the length of the span;
- Ratio of the static (ζ_S) and amplitude (ζ_a),vertical displacements to the length of the span;
- Angle of the static, (ϕ_S) and amplitude (ϕ_A) torsion of the span (in radians);
- Ratio of the death-weight to the imposed load γ.

Values of these parameters are given in tab. 2.

Tab. 2. Non-dimensional parameters

Footbridge	γ	Static displacements			Amplitudes		
		1000 ψ_S	1000 ξ_S	ϕ_S [rad]	1000 ψ_A	1000 ξ_A	ϕ_A [rad]
Rożnów	0.165	2.414	0.367	0.070	0.861	0.071	0.030
Tropie	0.101	1.010	0.353	0.059	0.063	0.009	0.004
Tylmanowa	0.312	0.047	0.000	0.000	0.018	0.066	0.000
Piwniczna	0.372	1.031	0.010	0.009	0.345	0.142	0.035

Conclusion

Basing upon obtained numerical results, following general conclusions can be formulated:

- For all footbridges ultimate limit state are fulfilled;
- For all footbridges horizontal and vertical stiffnesses, are suitable from statical point of view and enable to fullfill statical requirements according to standard [6];
- In the case of footbridges in Rożnów and Tropie static angle of tosion of the span is very big;
- For the footbridges in Rożnów, Tropie and Piwniczna dynamic serviceability limit state in horizontal direction is not fullfilled according to [7];
- Only footbridge in Tylmanowa fullfills all serviceability limit state;
- For all footbridges, no aerodynamical phenomenon (such as galloping, flutter, divergence) take place;
- Presented in this paper calculation results have been obtained taking into account only three the lowest symmetrical or quasi-symmetrical free vibration modes. In further aerodynamical calculations also the asymmetrical or quasi-asymmetrical free vibration modes should be taken into account.

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