Study of aerodynamical behaviour of different types of light footbridges against wind load

A. Flaga
Cracow University of Technology and Lublin Technical University, Poland
T. Michałowski
Cracow University of Technology, Cracow, Poland
G. Bosak
Cracow University of Technology, Cracow, Poland

ABSTRACT : An aerodynamical analysis of several existing light footbridges is presented. Different measures of making span footbridges stiff enough (trusses, inclined cable system, additional horizontal ropes) under wind load, is considered and analysed detaily. A model of wind load adopted in agreement with a quasi-steady concept takes into consideration not only unsteady air onflow but also motion of the structure itself. Wind load caused by vortices is neglected. In addition, it is assumed, that considered structures behave in a linear elastic way.

1. THEORETICAL APPROACH TO A PROBLEM

An analysys of four existing light footbridges is made with assumptions as follows :

- Wind action on pylons and cable systems of footbridges is of static-type;
- Wind action on span of footbridges is considered in accordance with a quasi-steady theory [1, 5].
- Linearised small vibrations of footbridges around their mean (static) position are considered.

Quasi-steady theory enables to consider several aerodynamic phenomena, such as galloping, special type of flutter, buffeting, and divergence. Three components of wind load include static and dynamic wind actions on span footbridges. These three components of wind load, i.e. drag force, lift force and aerodynamic moment, can be given by :

$$q = \rho(\bar{u} + u'_s)^2 / 2 \qquad (1)$$

$$W = (v'_s + \xi^* \sin\bar{\alpha} - \eta^* \cos\bar{\alpha})/(\bar{u} + u'_s) - [X_G \cos(\bar{\alpha} - \varepsilon) + Y_G \sin(\bar{\alpha} - \varepsilon)]/(\bar{u} + u'_s)]\varepsilon^* - \varepsilon \qquad (2)$$

$$w_x = q\,H\,[C_x + C_{xy}\,W] \qquad (3)$$

$$w_y = q\,H\,[C_y + C_{yx}\,W] \qquad (4)$$

$$w_m = q\,H^2\,[C_m + C_{mm}\,W] \qquad (5)$$

where : ρ - density of air; u, u', v', α, η, ξ, ε – as in fig. 1; H – characteristic diameter of cross-section; X_G and Y_G – coordinate of geometric centre of the outline curve of cros-ssection; C_x , C_{xy} , C_y , C_{yx} , C_m , C_{mm} – aerodynamic coefficients; □ – time averaged value; $□^*$ = d□/dt; □' – fluctuations; $□_s$ – spatially averaged value.

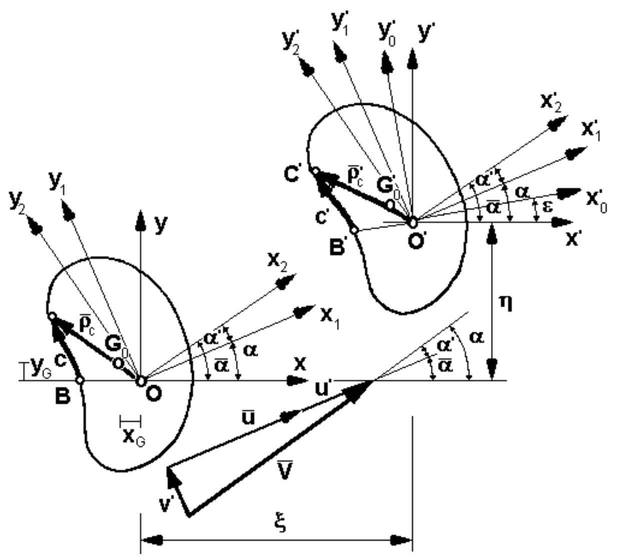

Fig. 1 Systems of coordinates and relations between different geometrical quantities in the case of a moving structure and an unsteady air onflow

2. MATHEMATICAL MODEL OF THREE DEGREES OF FREEDOM SYSTEM AS A SUBSTITUTIONAL SYSTEM OF EXISTING STRUCTURES

Spans of the censidered footbridges should be treated as a slender structure, which can be divided into m elements. Each of them is represented by the inner point k of the cross-section (see fig. 2).

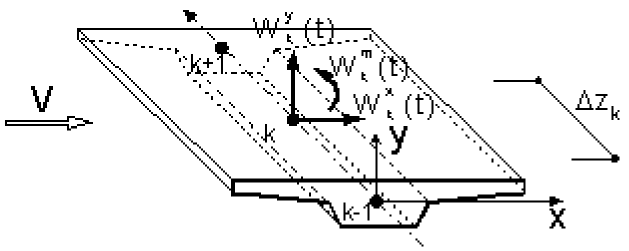

Fig. 2 A sector of span of bridge under wind load

Formulas (1-5), which are related to all points of span next are transformed to the global system of coordinates. It is assumed, that the structures behave in a linear elastic way. Their motion can be described by a matrix equation :

$$[M]\{r^{**}\} + [C]\{r^{*}\} + [K]\{r\}=\{w\} \quad (6)$$

The Bubnow-Galerkin's method is adopted. As a result, a system of many degrees of freedom is substituted by a representative system of three degrees of freedom. Motion of the substitutional system under wind load described by the formulaes (1-6) is given by :

$$M_j\Psi_j^{**}(t)+C_j\Psi_j^{*}(t)+K_j\Psi_j(t)= \quad (7)$$

$$=\sum_k^m \langle X^j\{A(t)\}_k^{\xi T}+Y^j\{A(t)\}_k^{\eta T}+\Phi^j\{A(t)\}_k^{\varepsilon T}\rangle[\chi(t)]_k$$

$$\{A(t)\}_j^{\xi T} = [C_xW_K(t),\ C_{xy}W_K(t)a_k(t),\ C_{xy}W_K(t)b_k(t),\ -C_{xy}W_K(t)c_k(t),\ -C_{xy}W_K(t)d_k(t),\ C_{xy}W_K(t)e_k(t),\ -C_{xy}W_K(t)] \quad (8)$$

$$\{A(t)\}_j^{\xi T} = [C_yW_K(t),\ C_{yx}W_k(t)a_k(t),\ C_{yx}W_k(t)b_k(t),\ -C_{yx}W_k(t)c_k(t),\ -C_{yx}W_k(t)d_k(t),\ C_{yx}W_k(t)e_k(t),\ -C_{yx}W_k(t),] \quad (9)$$

$$\{A(t)\}_j^{\varepsilon T} = [HC_mW_K(t),\ HC_{mm}W_K(t)a_k(t),\ HC_{mm}W_K(t)b_k(t),\ -HC_{mm}W_K(t)c_k(t),\ -HC_{mm}W_K(t)d_k(t),\ HC_{mm}W_K(t)e_k(t),\ -HC_{mm}W_K(t)] \quad (10)$$

$$\{\chi(t)\}^T{}_k=[1,\ 1,\ \xi^*(t)_k,\ \eta^*(t)_k,\ \varepsilon^*(t)_k,\ \varepsilon^*(t)_k\varepsilon(t)_k,\ \varepsilon(t)_k,] \quad (11)$$

$$W_k(t) = H\rho u^2\Delta z_k \quad (12)$$

$$u= u+ u' \quad (13)$$

$$a_k(t,z) = v'(t,z) / u(t,z) \quad (14)$$

$$b_k(t,z) = \sin\alpha(z) / u(t,z) \quad (15)$$

$$c_k(t,z) = \cos\alpha(z) / u(t,z) \quad (16)$$

$$d_k(t,z) = [X_G\ \cos\alpha(z) + Y_G \sin\alpha(z)] / u(t,z) \quad (17)$$

$$e_k(t,z) = [Y_G\ \sin\alpha(z) - Y_G \cos\alpha(z)] / u(t,z) \quad (18)$$

where : $\Psi_j(t)$ – j-th generalized coordinate; X^j, Y^j, Φ^j – translation and rotation coordinates of j-th free vibration mode of the point which represents k-th structural element; m- number of structural elements; M_j, C_j K_j – generalized mass, damping and stiffeness, Δz_k – length of k-th structural element.

The complex form of the nonlinear set of differential equations is resolved in a numerical way. The Newmark method of integration of the motion equations is adopted. The algorithm of the Newmark method is based on the own computer programme which gives the general displacements ξ, η, ε for each time step in k representative points for particular structural elements.

3. DESCRIPTION OF ANALYSED STRUCTURES

Among many means of making span of light footbridge stiff enough, three different metods are presented. First type of footbridge is a cable-ribbon suspension footbridge in Myślenice (A, see fig. 3). In this case transversal and torsional stiffness are ensured by two pairs of horizontal ropes, additionally attached to the span.

Second type of footbridge is suspension footbridge in Rożnów (B, see fig. 4) and in Tropie (C, see fig. 5). In these footbridges transversal and torsional stiffness are ensured mainly by inclined cable system.

Third type of footbridge is a cable-stayed footbridge (D, see fig. 6) in Rzeszów. Horizontal stiffness of this footbridge is mainly ensured by horizontal truss, what is the most often case encountered in practice. Torsional stiffness comes from two vertical trusses and stays.

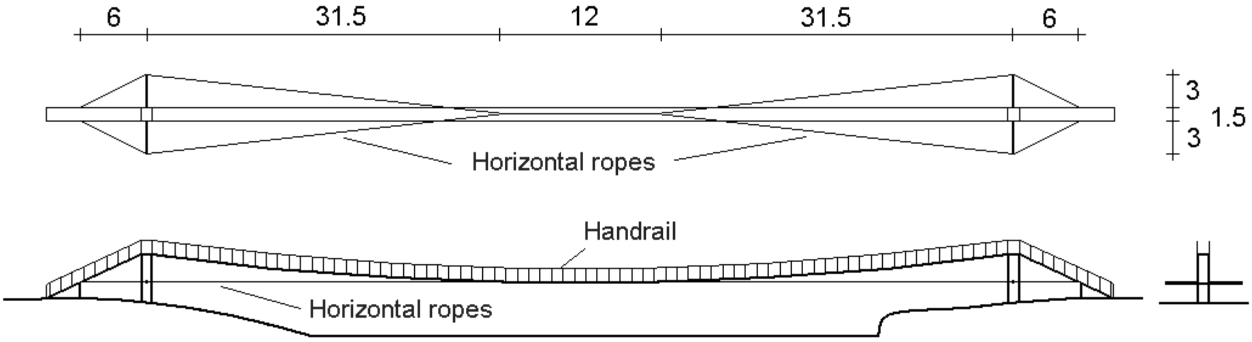

Fig. 3. View of footbridge A in Myślenice (dimension in meters)

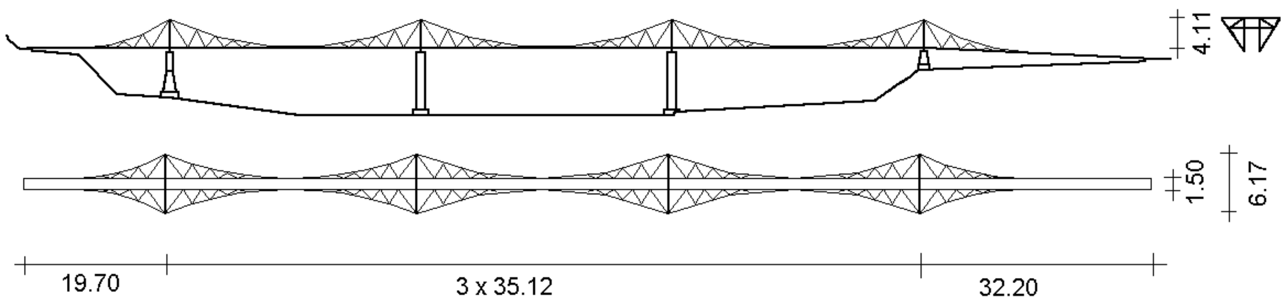

Fig. 4. View of footbridge B in Rożnów (dimension in meters)

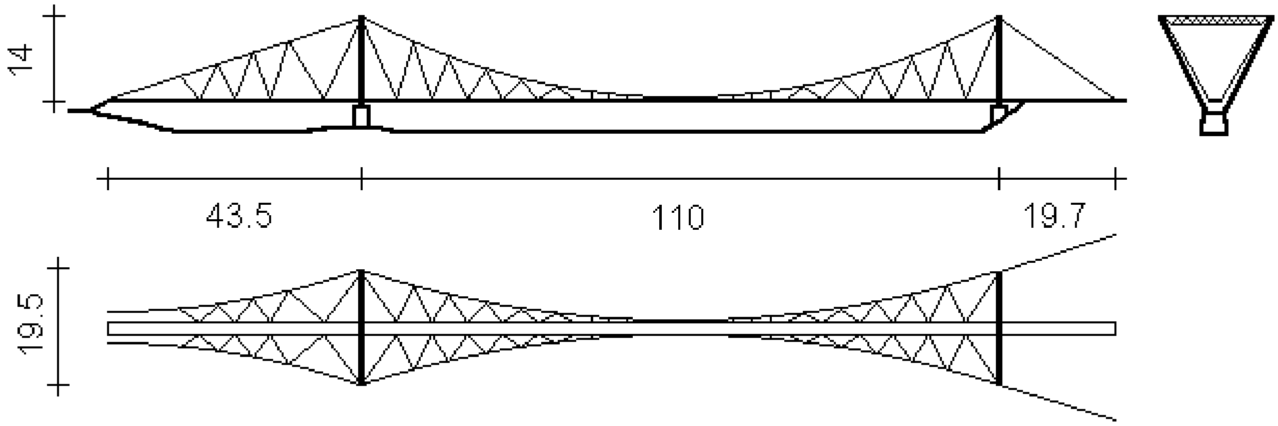

Fig. 5 View of footbridge C in Tropie (dimension in meters)

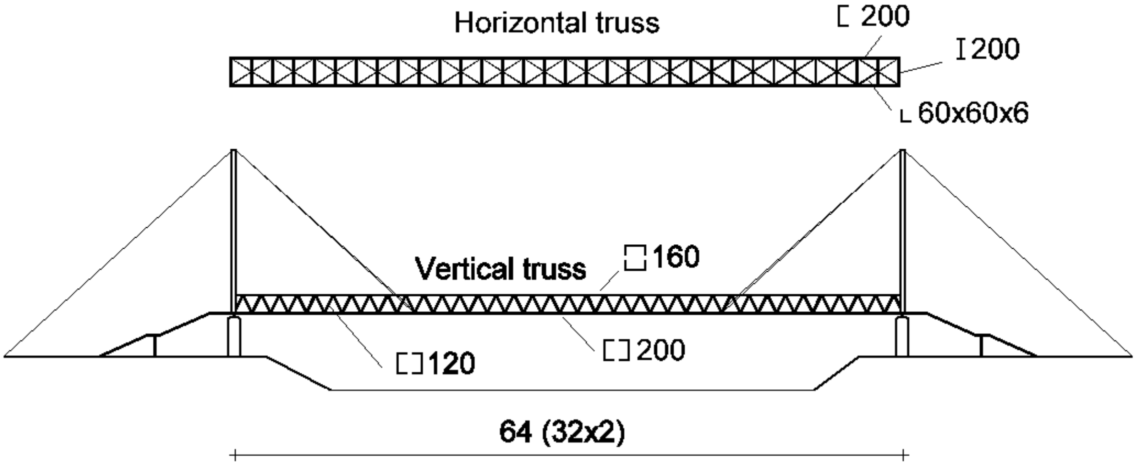

Fig. 6. View of footbridge D in Rzeszów (dimension in meters)

4. EXEMPLARY CALCULATION RESULTS

4.1. General assumptions

- Linearised small vibrations of footbridges, around their static position, are considered. This means, that vibrations of footbridges influenced by static component of wind load, come in question. Because of this, a character of vibration modes can be complicated and not so „clear" (vertical, horizontal etc.), but of a spatial character (quasi-vertical, quasi-symmetrical, etc.).
- With respect to horizontal, vertical and torsional directions, the lowest three symmetrical or quasi-symmetrical vibration modes, , are considered.

Exemplary choosen free vibration modes of the considered structures are presented in figs. 7 ÷ 14. Values of these and the others eigenfrequencies are given in tab. 1.

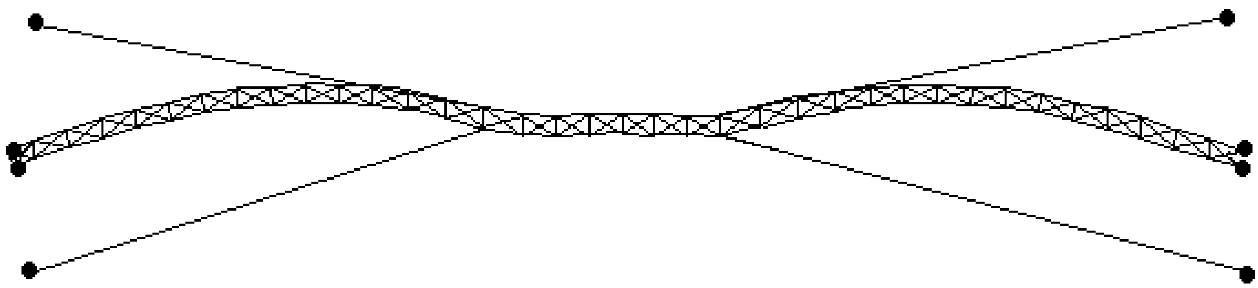

Fig. 7. I-st symmetrical vibration mode of footbridge A, plain view

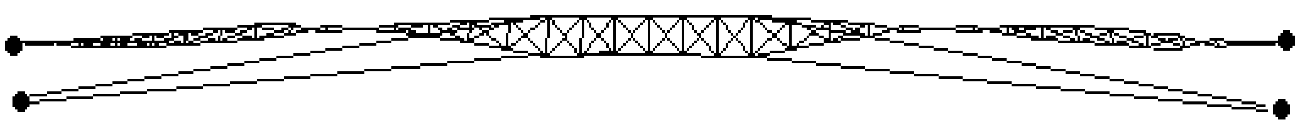

Fig. 8. II-nd symmetrical vibration mode of footbridge A, side view

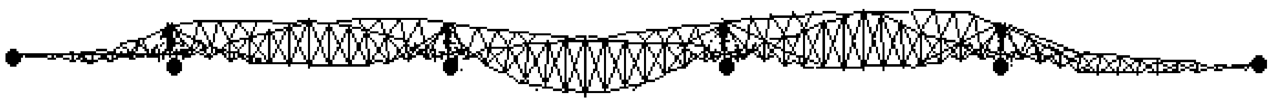

Fig. 9. I-st quasi-symmetrical vibration mode of footbridge B, side view

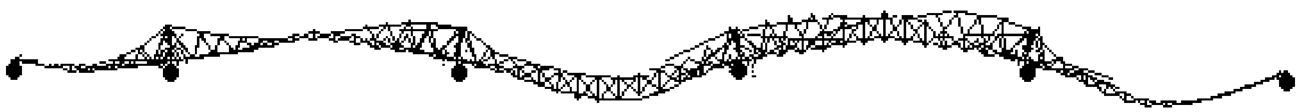

Fig. 10. II-nd quasi-symmetrical vibration mode of footbridge B, plain view

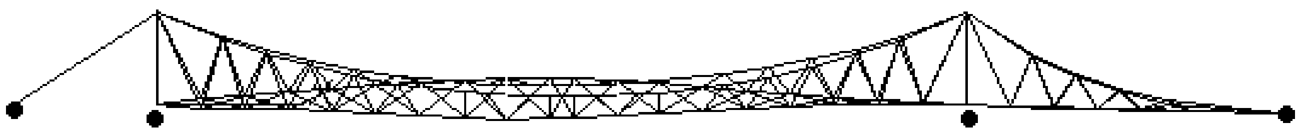

Fig. 11. I-st quasi-symmetrical vibration mode of footbridge C, side view

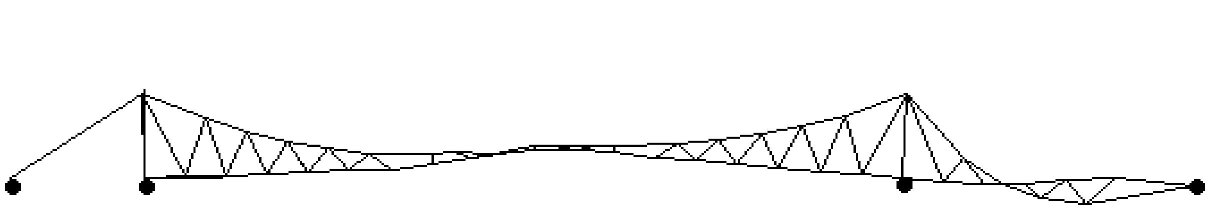

Fig. 12 II-nd quasi-symmetrical vibration mode of footbridge C, side view

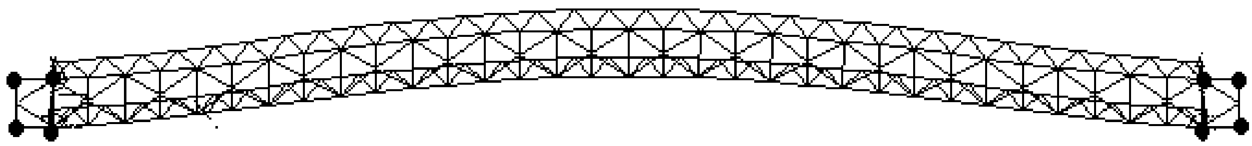

Fig. 13. I-st symmetrical vibration mode of footbridge D, plain view

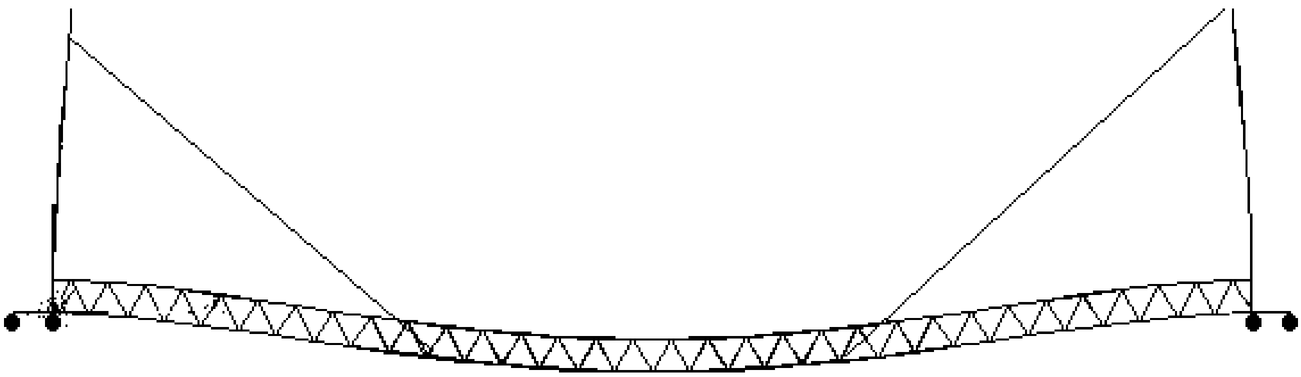

Fig. 14. II-nd symmetrical vibration mode of Footbridge D, side view

Table 1. Free vibration modes and eigenfrequencies of the footbridges

Footbridge	Frequencies [Hz] and modes									
A	0.923 I-st SH	1.004 I-st AV, I-st AT	1.049 I-st AH	1.072 I-st ST, I-st SV	1.152 II-nd SV	1.192 II-nd ST	1.334 II-nd AT	1.571 III-rd ST	1.655 III-rd AT II-nd AV	1.670 II-nd SH
B	1.185 I-st SH, I-st ST	1.321 I-st AH I-st AT	1.394 I-st SV	1.451 I-st AV	1.501	1.641	1.902	1.923	1.937	2.062
C	0.572 I-st SH I-stST	1.096 I-st AT	1.116 I-st SV	1.219 I-st AH	1.296 I-st AV	1.356	1.370	1.590	1.771	1.800
D	1.040 I-st SH	1.587 I-st SV	1.878 I-st ST	2.472 P	3.030 I-st AH	3.756 I-st AV	4.259 P	4.308 P	4.392 P	4.420 I-st AT

S – symmetrical or quasi-symmetrical mode,
A – antisymmetrical or quasi-antisymmetrical mode,
H – horizontal or quasi-horizontal mode,
V – vertical or quasi-vertical mode,
T – torsional or quasi-torsional mode,
P – deformation of pylons
Classification of the vibration modes of the footbridge B and C is complicated
Frequencies and modes considered in calculations.

4.2. Data assumed in calculations

- Wind characteristics :

- wind is simulated by WAWS method;
- von Kármán spectrum of wind is adopted;
- scale of turbulence is equal to 100 m.;
- power wind profile is assumed;
- mean wind velocity (10 m. above ground) u =10 m/s and 20 m/s.

Fig. 15. Shape of considering cross-sectiom

- Aerodynamical coefficients are assumed for a cross-section of span showed in fig. 15. This cross-section is similar to cross-sections of the considered footbridges, so the aerodynamical coefficients are as follows :
  $C_x = 0.214$,
  $C_{xy} = 0$,
  $C_y = -0.067$,
  $C_{yx} = 7.448$,
  $C_m = 0.012$,
  $C_{mm} = 0.206$.
- Logarythmic decrement of damping Δ = 0.060 for footbridge D and 0.040 for the others.

4.3 Respnse of the structures

For the each footbridge mean, maximum and r.m.s. value of horizontal, vertical and torsional displacements have been obtained. In analysis of aerodynamic behaviour of footbridges, several non-dimensional parametrs have been used:

- Ratio of the static (average; ψ_{st}), r.m.s. (ψ_{rms}), and maximum, (, ψ_{max}) horizontal displacement to the length of the span;
- Ratio of the static and maximum vertical displacements - η_{st}, η_{max} – are less than 0.0001;
- Angle of the static, (ε_{st}), r.m.s. (ε_{rms}), and maximum (ε_{max}) torsion of the span (in radians);
- Ratio of deathweight to the imposed load γ.

5. GENERAL REMARKS

General conclusions can be formulated as follow :

- All footbridges in horizontal direction are stiff enough;
- The parameter γ (see tab. 2) for footbridge D is about three times greater than for other foot-bridges;
- Among all considered footbridges, the para-meters ψ and ε reach their minimum for footbridge D;
- Vertical displacements induced by wind, compared to displacements induced by death-weight, could be neglected.

Moreover, calculation results and analysis of aerodynamical divergence of the footbridges permit to formulate following additional remarks :

- There is no phenomenon of aerodynamical divergence for footbridges B and D;
- This phenomenon took place in reality in the case of footbridge A without two horizontal ropes [3]. However, this phenomenon does not exist when additional horizontal ropes have been attached to the span;
- In case of footbridge C aerodynamical

Table 2 Values of dimensionless parameters for different footbridges

Parameters		Footbridges			
		A	B	C	D
u=10m/s	ψ_{st}	0.0006	0.0016	0.0025	0.0000
	ψ_{rms}	0.0000	0.0003	0.0000	0.0000
	ψ_{max}	0.0008	0.0027	0.0041	0.0000
	ε_{st}	0.0344	0.0492	0.0821	0.0077
	ε_{rms}	0.0007	0.0006	0.0000	0.0000
	ε_{max}	0.0631	0.0545	0.1123	0.0152
u=20m/s	ψ_{st}	0.0023	0.0057	0.0091	0.0009
	ψ_{rms}	0.0010	0.0020	0.0000	0.0000
	ψ_{max}	0.0039	0.0117	0.0162	0.0010
	ε_{st}	0.1535	0.1954	0.3642	0.0348
	ε_{rms}	0.0021	0.0011	0.0001	0.0000
	ε_{max}	0.2765	0.2641	0.4600	0.0704
γ		0.198	0.165	0.101	0.438

divergence theoretically could take place;

- Presented in this paper calculation results have been obtained taking into account only three the lowest symmetrical or quasi-symmetrical free vibration modes. In further aerodynamical calculations also the asymmetrical or quasi-asymmetrical free vibration modes should be taken into account.

6. REFERENCES

[1] Flaga A., 1994, Quasi-steady theory in aerodynamics of slender structures, SFB 151 Berichte Nr. 25,Wisserschaft-lichte Mitteilungen, Ruhr-Universität Bochum, 1994

[2] Flaga A., Flaga K., Michałowski T., 1996, Aerodynamical problems of cable-stayed and suspension bridges, (in Polish) Inżynieria i Budownictwo 9/1996, 508-516

[3] Flaga A., 1985 Analysis of horizontal stiffness for light suspension bridges, (in Polish), Proc. of the XXXI Conf. KILiW PAN i KN PZITB, Krynica 1985, vol. I 59-65

[4] Flaga A., Michałowski T., 1998 Aaerodynamic stability analysis of footbridge of an inclined cable system Proc. of the 2nd EECWE, Prague 1998, 97-104

[5] Flaga A., 1999, Semiempirical models of aerodynamic and aeroelastic phenomena of cable-stayed or suspension bridge spans, (in Polish), Proc. of the XLV Conf. KILiW PAN i KN PZITB, Krynica, 1999, vol. VI 69-86