AERODYNAMIC STABILITY ANALYSIS OF FOOTBRIDGE OF INCLINED CABLE SYSTEM

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SUMMARY

A mathematical model of aerodynamical behaviour of a light footbridge of an inclined cable system is presented in the paper. Basing on this model, following problems are considered detailedly : dependence of the angle of inclination of suspension cables on the torsional and transverse stiffness and on the critical velocity of aerodynamic divergence of the footbridge span; dependence of the angular frequency of linearized oscillation of the footbridge span on the angle of cable inclination and on the mean oncoming wind velocity.

INTRODUCTION

Light suspension bridges are often used as footbridges, foot-traffic bridges, bearing structures for pipelines, etc. One of the ways of making span bridge stiff enough is application of a suspension cable system in form of an inclined cable system. Such type of structure can be competitive along with other types of stiffening, for example horizontal truss or reinforced concrete plate. Inclinination of the suspension cables enables making footbridge very slender and light.

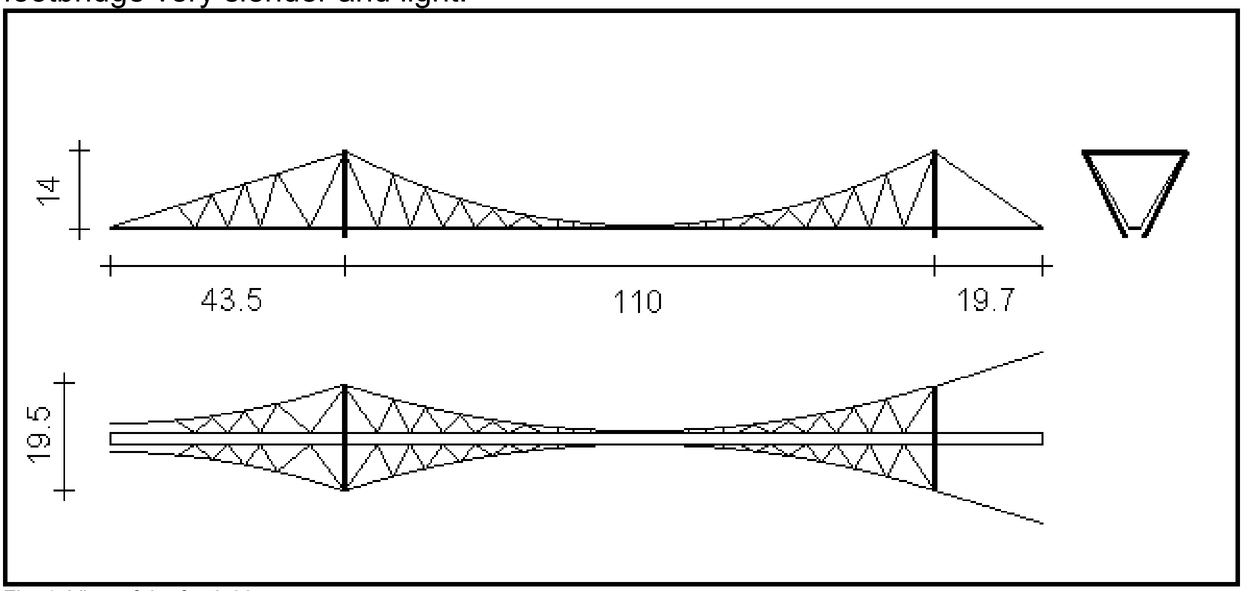

Fig. 1. View of the footbridge

Statical and aerodynamical analysis of such type of footbridge is presented below. This footbridge has no longitudinal beam of stiffening. It is made as an ansamble of steal traverses, subhanging to the inclined cables, loosely connected with a wooden pavement. Stiffness of the span is assured by inclinination of the cable system. Two pipelines are attached below the span. Thefoot bridge and cross-section of the span are showed in fig. 1 and 2.

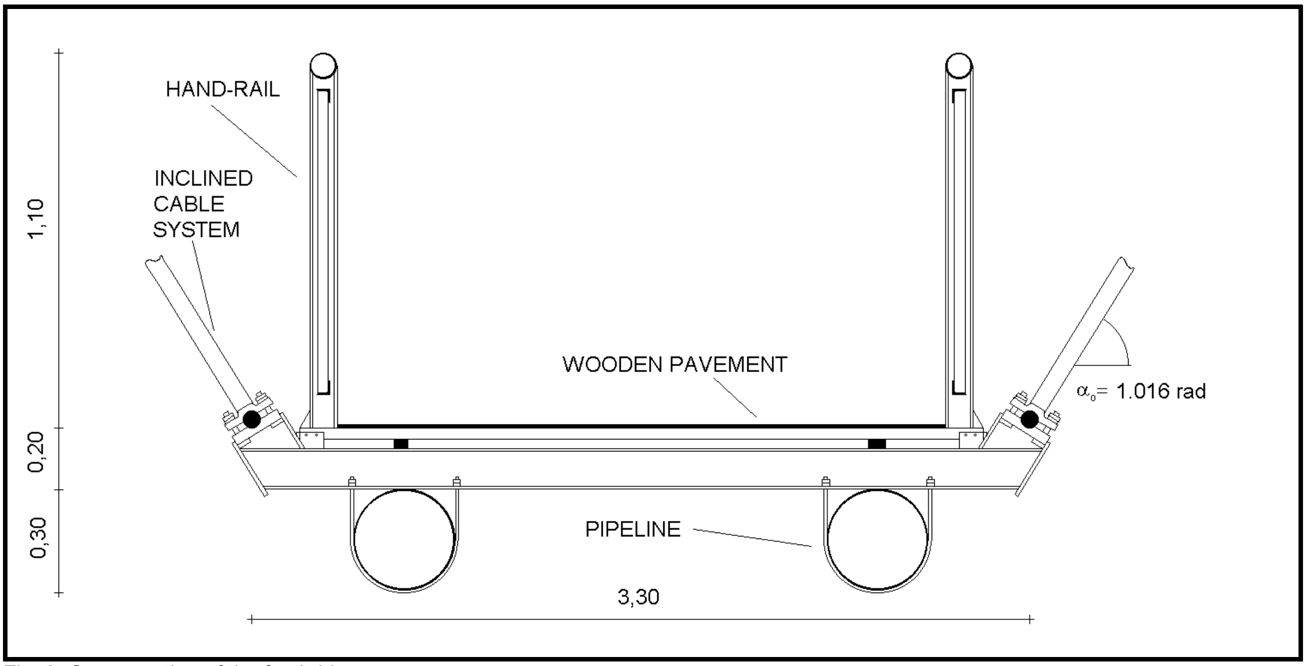

Fig. 2. Cross-section of the footbridge span

MATHEMATICAL MODEL OF THE PROBLEM

The analysis is realized at following assumptions :

- Longitudinal axis of the bridge span is horizontal
- Aerodynamical forces are dependent on the torsional angle θ (see fig. 3)
- Only average oncoming wind velocity is taken into considerations
- A segment of bridge "far" from the pylons is considered
- In statics, large displacements are considered (see fig. 5)
- In dynamics, small displacements around the static state of equilibrium are considered
- Span is very flexlible and has not enough stiffness to prevent aerodynamical divergence
- Flexible deformations of all bridge segments are neglected
- Mass of the considered segment of the span is accumulated on edges of the

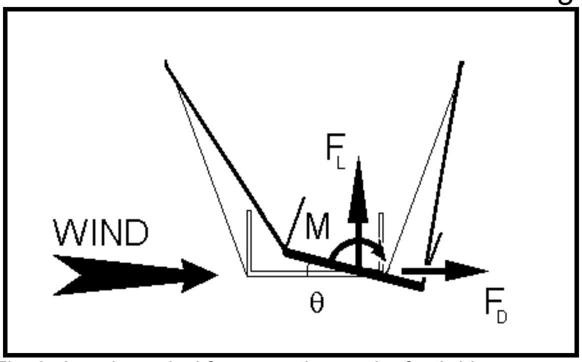

Fig. 3. Aerodynamical forces, acting on the footbridge span segment

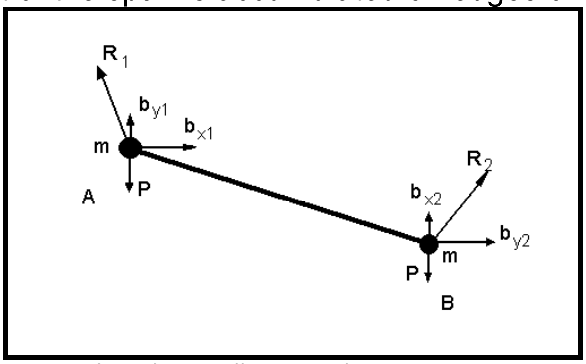

Fig. 4. Other forces, affecting the footbridge span segment

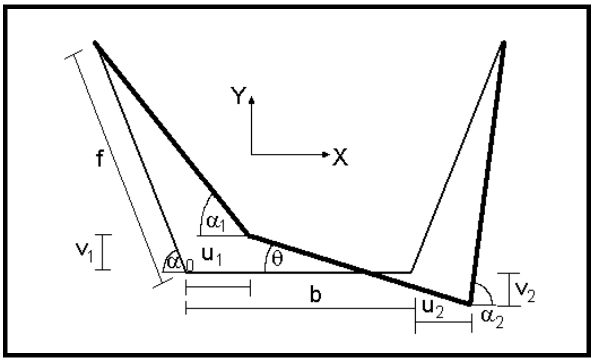

Fig. 5. Geometry of the deformed span segment

span (see fig. 4)

Forces affecting the considered fragment of the span, i.e. gravitional forces P = mg, aerodynamical forces F_L, F_D and the aerodynamical moment M, forces resulting from cables R_1, R_2 and (in dynamics) inertial forces b_{X1}, b_{Y1}, b_{X2}, b_{Y2} are showed in fig. 3 and 4. Figure 5 presents geometry of the deformed span.

STATIC AND DYNAMIC ANALYSIS OF THE SPAN SEGMENT

Static state of equilibrium is calculated from three equations of equilibrium : $\Sigma M_A = 0$, $\Sigma M_B = 0$, $\Sigma X = 0$:

$$R_2 = [\ M/b - 0.5(F_D \sin\theta + F_L \cos\theta - P \cos\theta)] / (\sin\alpha_2 \cos\theta + \cos\alpha_2 \sin\theta) \quad (1)$$

$$R_1 = [\ M/b + 0.5(F_D \sin\theta + F_L \cos\theta - P \cos\theta)] / (\cos\alpha_1 \sin\theta - \sin\alpha_1 \cos\theta) \quad (2)$$

$$F_D + R_2 \cos\alpha_2 - R_1 \cos\alpha_1 \quad (3)$$

Followig equations are assumed for aerodynamical forces :

$$F_D = Cd\ h\ \rho V^2/2 \quad (4)$$

$$F_L = Cl\ h\ \rho V^2/2 \quad (5)$$

$$M = Cm\ h\ b\ \rho V^2/2 \quad (6)$$

Aerodynamical coefficents in equations (4)-(6) must be determined from investigations in wind tunnel, or taken from publications. Typical cross-sections of bridge decks for which aerodynamical coefficients can be found in literature (for example [1], [2], [3], [4], [5]), are presented in fig. 6. It has been decided, that in further considerations

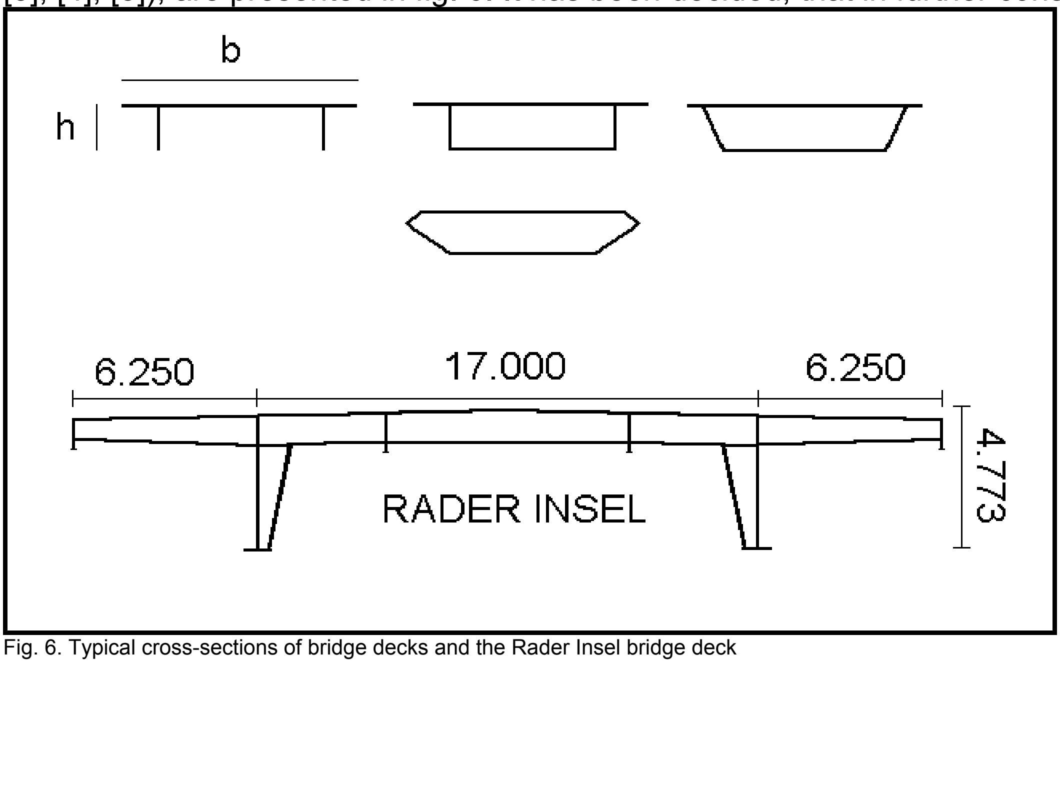

Fig. 6. Typical cross-sections of bridge decks and the Rader Insel bridge deck

aerodynamical coefficients as for Rader Insel bridge will be assumed. These aerodynamical coefficients are presented in fig. 7.

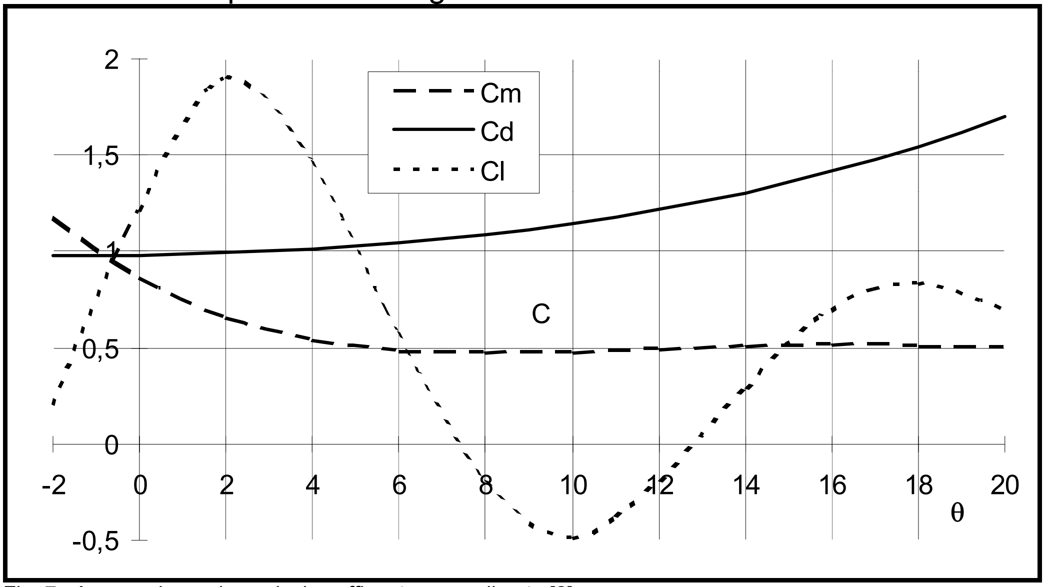

Fig. 7. Assumed aerodynamical coefficents, according to [2]

Besides, two geometrical equations are need for calculations (comp. fig. 5) :

$$\sin\theta = f(\sin\alpha_2 - \sin\alpha_1)/b \qquad (7)$$

$$\cos\theta = 1 + f(2\cos\alpha_0 - \cos\alpha_1 - \cos\alpha_2)/b \qquad (8)$$

These equations connect the displacements of the foot bridge span (see fig. 5).

Equations (1)-(8) present a system of eight equations of eigth unknows (R_1, R_2, $M(\theta)$, $F_L(\theta)$, $F_D(\theta)$, θ, α_1, α_2). Such a system of equations was resolved in a numerical way.

In dynamical analysis, new equations involving inertial forces (see fig. 4) are taken into account :

$$b_{X1} = -m\, u_1'' \qquad (9)$$

$$u_1 = f\,(\cos\alpha_1 - \cos\alpha_0) \qquad (10)$$

$$b_{Y1} = -m\, v_1'' \qquad (11)$$

$$v_1 = f\,(\sin\alpha_0 - \sin\alpha_1) \qquad (12)$$

$$b_{X2} = -m\, u_2'' \qquad (13)$$

$$u_2 = f\,(\cos\alpha_0 - \cos\alpha_2) \qquad (14)$$

$$b_{Y2} = -m\, v_2'' \qquad (15)$$

$$v_2 = f\,(\sin\alpha_0 - \sin\alpha_2) \qquad (16)$$

Additional equations of dynamical equilibrium are as follows :

$$R_1 := \frac{\frac{M}{b} - 0.5\left(F_D \sin\theta + F_L \cos\theta - P\sin\theta\right) - m\left[\frac{d^2}{dt^2}(v_2)\cos\theta + \frac{d^2}{dt^2}(u_2)\sin\theta\right]}{\sin\alpha_2 \cos\theta + \cos\alpha_2 \sin\theta} \qquad (17)$$

$$R_1 := \frac{\frac{M}{b} + 0.5\left(F_D \sin\theta + F_L \cos\theta - P\sin\theta\right) - m\left[\frac{d^2}{dt^2}(v_1)\cos\theta + \frac{d^2}{dt^2}(u_1)\sin\theta\right]}{\sin\theta \cos\alpha_1 - \cos\theta \sin\alpha_1} \qquad (18)$$

$$F_D + R_2\cos\alpha_2 - R_1\cos\alpha_1 - m(u_1'' + u_2'') \qquad (19)$$

Equations (4)-(8) remain unchanged. In dynamical analysis only small displacements are considered, i.e. equations (4)-(8) and (17)-(19) are solved using only linear parts of Taylor's series, with respect to θ or α_1. After transformations the following equation is obtained :

$$R\, d\theta + S\, d\theta'' + T\, d\theta\, d\theta'' = 0 \qquad (20)$$

If oscillations are small enough to neglect member ($T\, d\theta\, d\theta''$), linear equation of harmonic oscillation is obtained and then the angular frequency of oscillation is $\omega = \sqrt{(R/S)}$.

This model and solutions were presented in [6].

RESULTS OF NUMERICAL CALCULATIONS

Figure 8 presents the angle of span torsion θ as a function of the average wind velocity V.

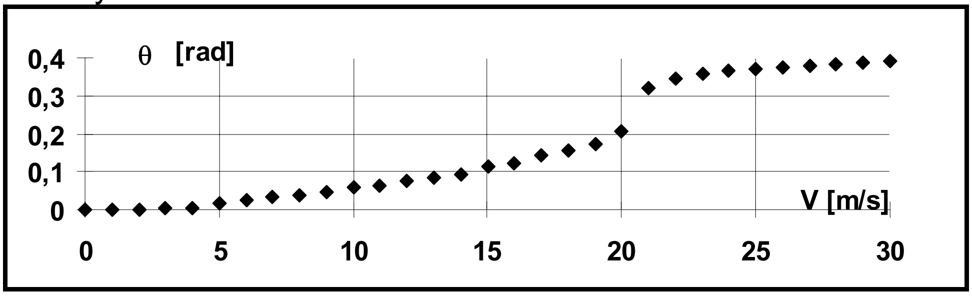

Fig. 8. Angle of span torsion θ as a function of average wind velocity V

A violent jump at approximately 20 m/s can be recognized as the phenomenon of aerodynamical divergence. Analysis of that phenomenon for different angles of inclination of the cable system α_0 shows, that the critical wind welocity of divergence V_{cr}^{Div} (see fig. 9) raches minimum approximately at 65^o. For angles α_0 greater than 85^o, the critical velocity of divergence is greater than 30 m/s.

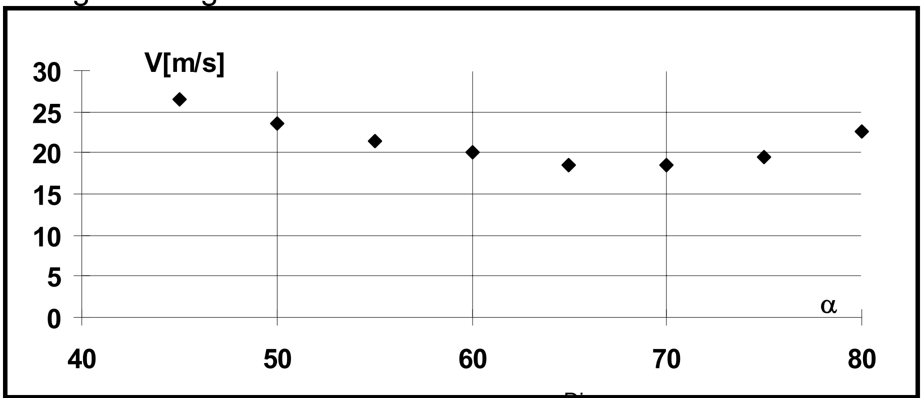

Fig. 9. Dependence of the critical velocity of divergence V_{cr}^{Div} on the angle of inclinination of the cable system α_0

Torsional and transverse stiffness of the span is dependent on the angle α_0. Figures

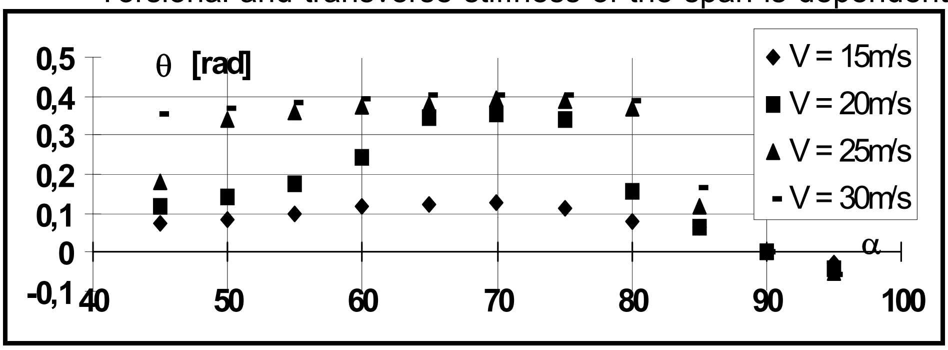

Fig. 10. Dependence of the torsional angle of the span θ on the average wind velocity V and on the angle of inclination of the cable system α_0

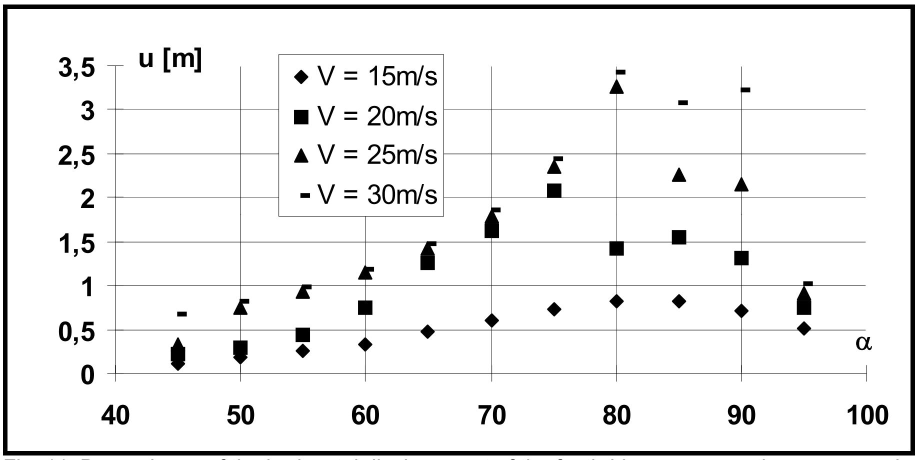

Fig. 11. Dependence of the horizontal displacement of the footbridge span u on the average wind velocity V and on the angle of inclination of the cable system α_0

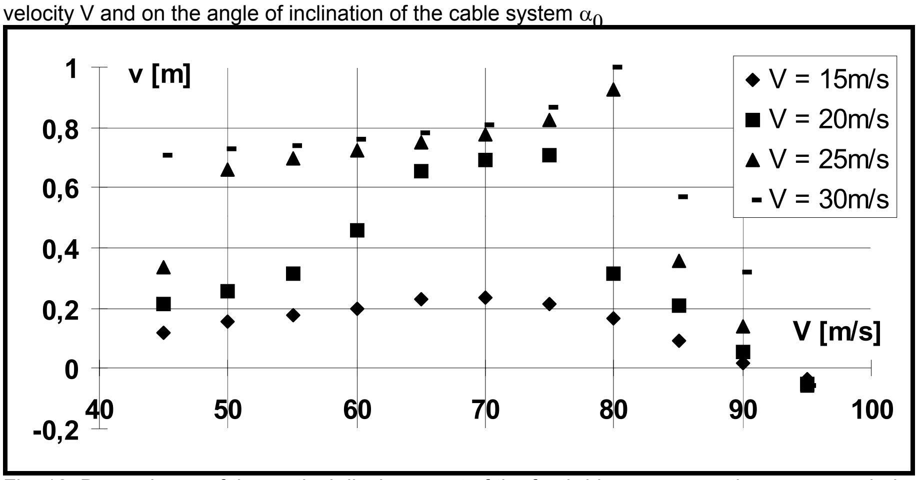

Fig. 12. Dependence of the vertical displacement of the footbridge span v on the average wind velocity V and on the angle of inclination of the cable system α_0

10, 11 and 12 show, consequently, the dependence of the torsional angle of the span θ, horizontal (u) and vertical (v) dispalcements of the point A (see fig. 4) on the angle of inclination of the cable system α_0 for different average wind velocities V.

The torsional angle θ and the displacements u and v reach minimum for an angle $\alpha_0 = 95^o$.

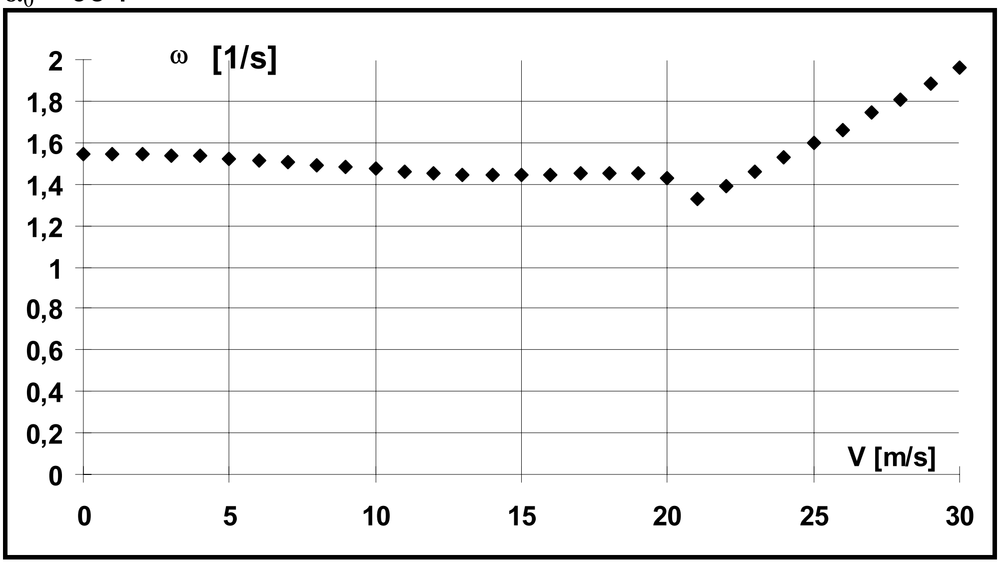

Fig. 13. Dependence of the angular frequency of the linaerized oscillation of the span segment ω on the average wind velocity V

When the torsional angle θ, and the dispalcements u and v are konwn, it is possible to control the serviceability conditions. For example, foot traffic can exsist when $\theta < 0.175$ rad., i.e. 10^o.

Dependence of the angular frequency of the linearized oscillation of the span segment ω on the average wind velocity V is presented in fig. 13.

Aerodynamic divergence is revealed in dynamical analysis, too. The diagram in fig. 14 breaks locally for the critical velocity of divergence $V_{cr}^{Div} = 20$ m/s.

In expression (20), the coefficent R is connected with the stiffness of the segment span, and the coefficent S is connected with the inertia of the segment span bridge. S is almost constant for all wind velocities, so R is depended on wind velocity. The critical velocity of divergence is such a velocity, for which deformation of the span member causes violent change of span sitffness.

Figure 15 presents dependence of the angular frequency of oscillation ω on the average wind velocity V and on the angle of inclination of the cable system α_0.

Figure 16 presents dependence of the angular frequency ω for the angle of inclined

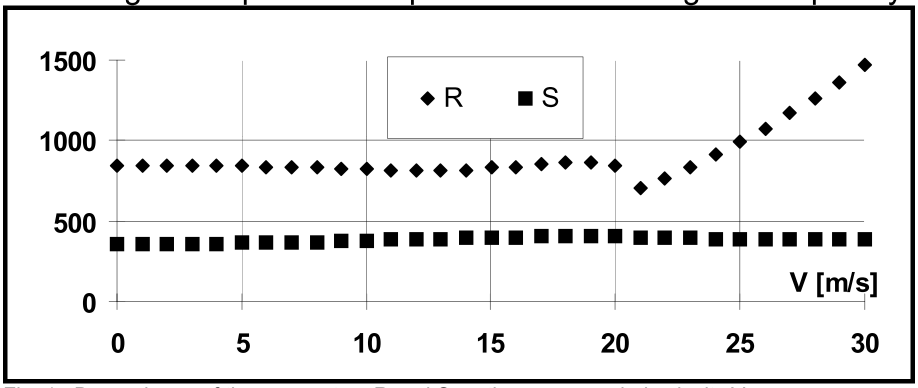

Fig. 14 Dependence of the parameters R and S on the average wind velocity V

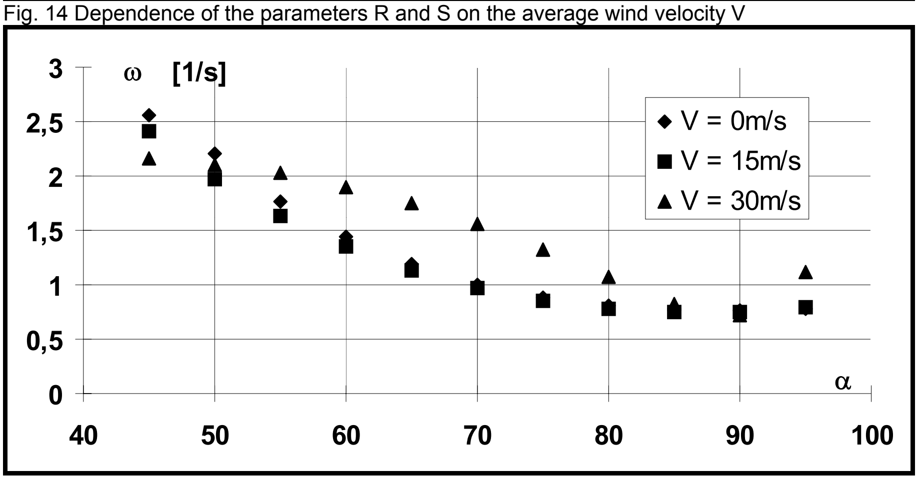

Fig. 15. Dependence of angular frequency of oscillation ω on the average wind velocity V and on the angle of inclination of cable system α_0

cable system $\alpha_0 = 90^o$ on the average wind velocity V. For such an angle of the inclined cable system, the behaviour of the considered span segment can be evaluated as for a mathematical pendulum. Length of the pendulum is equal f (see fig. 5).

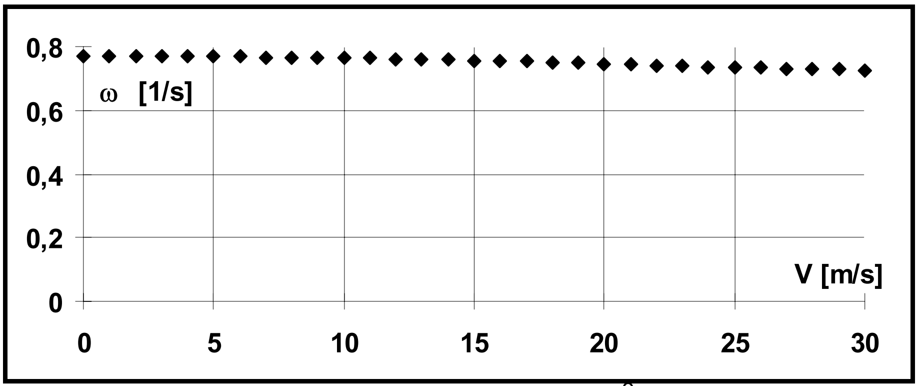

Fig. 16. Dependence of the angular frequency of span for $\alpha_0 = 90^o$ on the average wind velocity V

CONCLUSIONS

Analysis of the obtained results enables to present the following conclusions :

- The assumed mathematical model permits to describe the phenomenon of aerodynamical divergence
- Torsional and transverse stiffness is dependent on the angle of the inclined cable system α_0
- For small wind velocities (V < 15 m/s)serviceability conditions are fulfilled
- Minimum displacements appear at angle $\alpha_0 = 95^o$
- For angle $\alpha_0 = 90^o$ behaviour of such a type of structure can be evaluated as a mathematical pendulum
- This way of making span stiff enough can be cmpetitive along with other ways of stiffening, for instance horizontal truss.

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